RAY'S MODERN PRACTICAL ARITHMETIC

AMERICAN BOOK COMPANY

Miss —

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St. Agnes Academy

ECLECTIC EDUCATIONAL SERIES

RAY'S MODERN

PRACTICAL ARITHMETIC

A REVISED EDITION OF

RAY'S PRACTICAL ARITHMETIC

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PREFACE

THE purpose of this revision of Ray's "New Practical Arithmetic" is to adapt the book more closely to modern methods of teaching Arithmetic, without removing any of the features that have so long held the favor of thousands of teachers and pupils throughout the country. The revision, therefore, provides for the enrichment of some subjects, like Practical Measurements, Bills and Accounts, etc., which receive much consideration in the modern school curriculums, the abridgment of others of waning importance, and the total omission of those that have become obsolete.

The philosophical method of the original edition, however, remains unchanged. Every principle is clearly explained by an analysis or solution of a simple example from which a rule is derived. The application of the rule to the solution of problems of gradually increasing difficulty completes the presentation of the subject.

The exercises have been constructed with a view to affording the mental discipline necessary to strengthen the reasoning power and to giving the pupil a mastery over the problems that are sure to present themselves in the common walks of life. They are founded on considerations both of psychological pedagogy and of practical utility.

Miss —

Clare —

Smith —

St. Agnes Academy

MODERN PRACTICAL ARITHMETIC

NOTATION AND NUMERATION

1. A **unit** is a single thing of any kind; as one, one apple, one dollar, one pound.

A **number** consists of one or more units; as one, five, seven cents, nine men.

Arithmetic is the science that treats of numbers and the methods of using them.

Numbers may be expressed by *words*, by *figures*, or by *letters*.

A **system of notation** is a method of expressing numbers by figures or by letters.

Two systems of notation are in use, the *Arabic* and the *Roman*. The Arabic system is used in all our arithmetical calculations.

THE ARABIC SYSTEM OF NOTATION

2. To express numbers, the Arabic notation employs ten characters, called *figures;* namely, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The first nine numbers are represented by the first nine figures, and all the other numbers by combinations of the ten figures.

NOTE.—The Arabic system of notation was so called because its characters were supposed to have been introduced into Europe by the Arabians; but it is now generally acknowledged that they originated in India. It is also called the **decimal system** and the **common system**.

The **order** of a figure is the place it occupies in a number.

Units of the first order, or units

3. A unit or single thing is *one*, written 1.
One unit and one more are *two*, written 2.
Two units and one more are *three*, written 3.
Three units and one more are *four*, written 4.
Four units and one more are *five*, written 5.
Five units and one more are *six*, written 6.
Six units and one more are *seven*, written 7.
Seven units and one more are *eight*, written 8.
Eight units and one more are *nine*, written 9.

These nine characters are called **significant** figures, because they denote something.

The character 0, called *naught*, *cipher*, or *zero*, stands for *nothing;* its use is to fill vacant orders.

When a figure stands alone or in the first place at the right of a number, it represents one or more *units of the first order*. Units of the first order are called simply *units;* and the place they occupy is called the *units' place*.

Units of the second order, or tens

4. Nine units and one more are called *ten*. Ten is represented also by the figure 1 written in the *second place* from the right and 0 in the units' place.

One ten is written 10.
Two tens are *twenty*, written 20.
Three tens are *thirty*, written 30.
Four tens are *forty*, written 40.
Five tens are *fifty*, written 50.
Six tens are *sixty*, written 60.
Seven tens are *seventy*, written 70.
Eight tens are *eighty*, written 80.
Nine tens are *ninety*, written 90.

When a figure in a number stands in the second place from the right, it represents one or more *units of the second order.*

Units of the second order are called *tens;* and the place they occupy is called the *tens' place.*

Tens and units

5. The numbers between 10 and 20, 20 and 30, etc., are expressed by representing the tens and units of which they are composed.

One	ten	and	one	unit	are	*eleven,*	written	11.
One	ten	and	two	units	are	*twelve,*	written	12.
One	ten	and	three	units	are	*thirteen,*	written	13.
One	ten	and	four	units	are	*fourteen,*	written	14.
One	ten	and	five	units	are	*fifteen,*	written	15.
One	ten	and	six	units	are	*sixteen,*	written	16.
One	ten	and	seven	units	are	*seventeen,*	written	17.
One	ten	and	eight	units	are	*eighteen,*	written	18.
One	ten	and	nine	units	are	*nineteen,*	written	19.
Two	tens	and	one	unit	are	*twenty-one,*	written	21.
Two	tens	and	two	units	are	*twenty-two,*	written	22.

Write:

1. Twenty-three; twenty-four; twenty-five; twenty-six; twenty-seven; twenty-eight; twenty-nine.

2. Thirty-seven; forty-two; fifty-six; sixty-nine; seventy-three; eighty-seven; ninety-four.

3. Eighty-three; forty-five; ninety-nine; fifty-one; thirty-six; seventy-eight; sixty-two.

4. Fifty-five; ninety-three; eighty-one; sixty-seven; forty-nine; seventy-four; thirty-eight.

5. Seventy-six; forty-four; eighty-two; fifty-seven; thirty-five; ninety-one; sixty-three.

Read:

1.	71	32	53	84	65	46	97
2.	58	34	79	66	41	85	92
3.	75	43	88	61	59	33	95
4.	39	72	54	86	47	98	64
5.	68	77	31	89	52	96	48

Units of the third order, or hundreds

6. Ten tens are *one hundred;* it is represented by the figure 1 written in the *third order*, the orders of tens and units being each filled with a cipher.

One hundred is written 100.
Two hundred is written 200.
Three hundred is written 300.
Four hundred is written 400.
Five hundred is written 500.
Six hundred is written 600.
Seven hundred is written 700.
Eight hundred is written 800.
Nine hundred is written 900.

Units of the third order are called *hundreds;* and the place they occupy is called the *hundreds' place.*

Hundreds, tens, and units

7. The numbers between 100 and 200, 200 and 300, etc., are expressed by representing the hundreds, tens, and units of which they are composed.

One hundred and one unit is written 101, and read *one hundred one.*

One hundred and one ten is written 110, and read *one hundred ten.*

One hundred, one ten, and one unit is written 111, and read *one hundred eleven.*

One hundred and two tens is written 120, and read *one hundred twenty.*

One hundred, two tens, and five units is written 125, and read *one hundred twenty-five.*

NOTE.—Do not read the word *and* between hundreds and tens or hundreds and units. Read one hundred fifteen, one hundred two, *not* one hundred *and* fifteen, one hundred *and* two.

Write:

1. One hundred thirty; one hundred forty; one hundred fifty; one hundred sixty; one hundred seventy; one hundred eighty.

2. One hundred twenty-three; four hundred fifty-six; seven hundred eighty-nine; one hundred forty-seven; two hundred fifty-eight; three hundred sixty-nine.

3. One hundred two; three hundred forty-five; six hundred seventy-eight; two hundred thirty-four; five hundred sixty-seven; eight hundred ninety.

4. Four hundred fifty-three; seven hundred eighty-six; nine hundred twelve; two hundred thirty; four hundred fifty; six hundred seventy.

5. One hundred fifty-three; four hundred eighty-six; seven hundred twenty-nine; one hundred three; four hundred six; seven hundred nine.

Read:

1.	210	320	430	540	650	760
2.	213	546	879	417	528	639
3.	201	435	768	324	657	980
4.	543	876	192	329	548	765
5.	513	846	279	301	604	907

Units of higher orders

8. Ten hundreds are *one thousand;* it is represented by 1 in the fourth order; thus, 1000.

Ten thousands form a unit of the *fifth order;* thus, 10000; one hundred thousands, a unit of the *sixth order;* thus, 100000, etc.

Invariably, *ten units of any order make a unit of the next higher order.*

The names of the first nine orders may be learned from the following table:

Table of Orders

9th	8th	7th	6th	5th	4th	3d	2d	1st
Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units

9. The **value of a figure** depends upon the place it occupies. Thus, the value of 5 in units' order is 5; of 5 in tens' order, 50; of 5 in hundreds' order, 500. The value of a figure is increased tenfold by removing it one place to the left, and decreased tenfold by removing it one place to the right.

Grouping of orders into periods

10. For convenience in writing and reading numbers, the different orders are grouped into **periods** of three orders each.

Note. — A number is *pointed off* into periods of three figures each by commas.

The first three orders — units, tens, hundreds — constitute the first, or *units'* period.

The second group of three orders — thousands, ten-thousands, hundred-thousands — constitutes the second, or *thousands'* period.

The third group of three orders constitutes the third, or *millions'* period.

The periods from the first to the twelfth, inclusive, may be learned from the following table :

TABLE OF PERIODS

No.	Name	No.	Name
First	Units	Seventh	Quintillions
Second	Thousands	Eighth	Sextillions
Third	Millions	Ninth	Septillions
Fourth	Billions	Tenth	Octillions
Fifth	Trillions	Eleventh	Nonillions
Sixth	Quadrillions	Twelfth	Decillions

The grouping of the orders into periods is shown as follows :

NUMERATION TABLE

5th Trillions			4th Billions			3d Millions			2d Thousands			1st Units		
Hundred-trillions	Ten-trillions	Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units

It is plain that ***each period is composed of units, tens, and hundreds of that period.***

11. To write numbers in the Arabic system:

1. Write six hundred fifty-four *trillion* eight hundred twenty-one *billion* nine hundred seventy-eight *million* two hundred fifty-four *thousand* three hundred fourteen.

Trillions			Billions			Millions			Thousands			Units		
6	5	4,	8	2	1,	9	7	8,	2	5	4,	3	1	4.
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units

Rule.—***Begin at the left, and write each period as a number composed of hundreds, tens, and units—filling the vacant orders with ciphers.***

NOTE.—In the left-hand period, however, when the hundreds or the hundreds and tens are wanting, the vacant orders are not to be filled with ciphers.

Write:

2. Two thousand; thirty thousand; four hundred thousand.

3. Five million; sixty million; seven hundred million.

4. Eight billion; ninety billion; one hundred billion.

5. One thousand two hundred; two thousand one hundred.

6. Three thousand four hundred fifty; six thousand seven hundred eighty-nine.

7. Twelve thousand three hundred forty-five.

8. Six hundred seventy-eight thousand nine hundred twelve.

9. One million three hundred fifty-seven thousand nine hundred twenty-four.

10. Sixty-eight million one hundred forty-three thousand seven hundred ninety-two.

11. One thousand one; one thousand ten; one thousand one hundred.

12. One thousand one hundred one; one thousand one hundred ten; one thousand one hundred eleven.

13. Two thousand three; four thousand fifty.

14. Forty-five thousand twenty-six.

15. Eighty thousand two hundred one.

16. Ninety thousand one.

17. Four hundred ten thousand two hundred five.

18. One hundred thousand ten.

19. Three million seventy thousand five hundred nine.

20. Forty-five million eighty-three thousand twenty-six.

21. Nine hundred nine million ninety thousand.

22. Seven hundred million ten thousand two.

23. Forty billion two hundred thousand five.

24. Seven hundred twenty-six billion fifty million one thousand two hundred forty-three.

25. Eighty billion seven hundred three million five hundred four.

12. Numeration is the reading of numbers when expressed according to a system of notation.

To read numbers in the Arabic system:

1. Read 654821978254314.

Trillions			Billions			Millions			Thousands			Units		
6	5	4,	8	2	1,	9	7	8,	2	5	4,	3	1	4.
Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units	Hundreds	Tens	Units

Rule. — 1. *Begin at the right, and point off the number into periods of three figures each.*

2. *Begin at the left, and read each period as a number composed of hundreds, tens, and units, giving the name of the period.*

Notes. — 1. The left-hand period will sometimes contain but one or two figures.

2. It is customary to omit the name of the units' period.

Read:

2. 41582; 763491; 2519834; 375486921; 4923176358.
3. 37584216974; 432685729145; 6253971438267.
4. 1300; 2540; 6070; 8009; 13200; 1005.
5. 682300; 8600050; 3040; 50004; 704208.
6. 7085; 62001; 400009; 2102102; 9001003.
7. 130670921; 6900702003; 23004090701; 9420163070.
8. 570000010326049; 103478511992485.
9. 900020108000507; 800820020802008.

THE ROMAN SYSTEM OF NOTATION

13. To express numbers, the Roman notation uses seven letters as follows: —

Letters . .	I	V	X	L	C	D	M
Values . .	1	5	10	50	100	500	1000

NOTE. — The Roman system of notation is so called because it was the method of expressing numbers used by the ancient Romans. It is now used to mark the chapters of books, the dial plates of clocks, etc.

Principles. — I. *Repeating a letter repeats its value.*

Thus, II represents two; III, three; XX, twenty; XXX, thirty; CC, two hundred; CCC, three hundred; MM, two thousand.

NOTE. — The letters V, L, and D are never repeated.

II. *When a letter is placed after another of greater value, their values are to be united.*

Thus, XI represents eleven; VII represents seven; LXXX represents eighty.

III. *When a letter is placed before another of greater value, its value is to be subtracted from that of the greater.*

Thus, IV represents four; IX, nine; XC, ninety; CD, four hundred; CM, nine hundred.

IV. *When a letter is placed between two letters, each of greater value, its value is to be subtracted from the sum of the other two.*

Thus, XIV represents fourteen; XIX, nineteen; XXIV, twenty-four; CXL, one hundred forty.

V. *A bar placed over a letter multiplies its value by* 1000.

Thus, $\overline{\text{V}}$ represents five thousand; $\overline{\text{X}}$, ten thousand; $\overline{\text{L}}$, fifty thousand; $\overline{\text{M}}$, one million; $\overline{\text{CD}}$, four hundred thousand.

14. The following table shows the method of combination:

I	1	XV	15	LX	60	MC	1100
II	2	XVI	16	LXX	70	MCC	1200
III	3	XVII	17	LXXX	80	MCCC	1300
IV	4	XVIII	18	XC	90	MCD	1400
V	5	XIX	19	C	100	MD	1500
VI	6	XX	20	CC	200	MDC	1600
VII	7	XXI	21	CCC	300	MDCC	1700
VIII	8	XXII	22	CD	400	MDCCC	1800
IX	9	XXIV	24	D	500	MCM	1900
X	10	XXVI	26	DC	600	MM	2000
XI	11	XXIX	29	DCC	700	MMM	3000
XII	12	XXX	30	DCCC	800	$\overline{\text{V}}$	5000
XIII	13	XL	40	CM	900	$\overline{\text{CM}}$	900,000
XIV	14	L	50	M	1000	$\overline{\text{M}}$	1,000,000

Read:

1.	2.	3.
XXIII	CLVI	MLXVI
XXXIX	CCXLIV	MDCCLXXVI
XLI	CDI	MDCCCLXIII
LXVI	DCIX	MDLXXXVIII
LXXIV	DCCCIV	MDCXX
XCVIII	MIX	MCMII

Write in Roman Notation:

4.	5.	6.	7.
23	46	73	94
57	101	106	117
109	246	309	1010
199	859	975	410
734	1119	1285	800
1066	1572	1812	1814
1492	1431	1850	1704
1776	1598	1793	1588
1800	1215	1527	1903

ADDITION

15. **1.** If you have 2 cents and find 3 cents, how many will you then have? *Ans.* 5 cents.

Why? Because 2 cents and 3 cents are 5 cents.

2. I spent 12 cents for a slate, and 5 cents for a copy-book. How many cents did I spend?

Ans. 17 cents. Why?

3. John paid 6 cents for an orange, 7 cents for pencils, and 9 cents for a ball. How many cents did all cost?

Ans. 22 cents. Why?

4. Joseph paid 5 cents for a daily paper, 10 cents for a weekly paper, 25 cents for a monthly magazine, 30 cents for a book of poems, and 40 cents for a novel. How much did he spend? *Ans.* 110 cents.

16. The operation in these examples is termed *addition*.

Addition is the process of uniting two or more numbers into one number.

The number obtained by addition is the **sum** or **amount.**

17. The **sign of addition** (+), called *plus*, means *more;* when placed between two numbers, it shows that they are to be added. Thus, $4 + 2$ means that 4 and 2 are to be added.

The **sign of equality** (=) denotes that the quantities between which it stands are equal. Thus, the expression $4 + 2 = 6$ means that the sum of 4 and 2 is 6. It is read, 4 *plus* 2 *equals* 6.

Addition Table

$2+0=2$ $2+1=3$ $2+2=4$ $2+3=5$ $2+4=6$ $2+5=7$ $2+6=8$ $2+7=9$ $2+8=10$ $2+9=11$	$3+0=3$ $3+1=4$ $3+2=5$ $3+3=6$ $3+4=7$ $3+5=8$ $3+6=9$ $3+7=10$ $3+8=11$ $3+9=12$	$4+0=4$ $4+1=5$ $4+2=6$ $4+3=7$ $4+4=8$ $4+5=9$ $4+6=10$ $4+7=11$ $4+8=12$ $4+9=13$	$5+0=5$ $5+1=6$ $5+2=7$ $5+3=8$ $5+4=9$ $5+5=10$ $5+6=11$ $5+7=12$ $5+8=13$ $5+9=14$
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18. When the sum of the figures in a column does not exceed 9, it is written under the column added.

1. I own three tracts of land; the first contains 240 acres; the second, 132 acres; the third, 25 acres. How many acres do all contain?

SOLUTION. — Since only units of the same order can be added, write units of the same order in the same vertical column, so that the figures to be added may be in the most *convenient position*.

Beginning at the right, find the sum of the units, $5+2$, and write this sum, 7, under the units' column. Next find the sum of the tens, $2+3+4$, and write this sum, 9, under tens' column. Next find the sum of the hundreds, $2+1$, and write the sum, 3, under hundreds' column.

OPERATION

240	acres.
132	acres.
25	acres.
397	acres.

NOTE. — In adding, name results only. Thus in the tens' column say 2, 5, 9, *not* 2 and 3 are 5 and 4 are 9.

2. I owe one man \$210, another \$142, and another \$35. What is the sum of my debts? \$387.

3. Find the sum of 4321, 1254, 3120. 8695.

4. Find the sum of 50230, 3105, 423. 53758.

19. When the sum of the figures in the column exceeds 9, *two* or *more* figures are required to express it.

1. Add the numbers 3415, 503, 1870, and 922.

SOLUTION. — Write units of the same order in the same column. The sum of the units, $2 + 3 + 5, = 10$ units, or 1 ten and 0 units. Write the 0 under units' column, and add the 1 to tens' column. The sum of the tens, $1 + 2 + 7 + 1, = 11$ tens, or 1 hundred and 1 ten. Write 1 under tens' column, and add the second 1 to hundreds' column. The sum of the hundreds, $1 + 9 + 8 + 5 + 4, = 27$ hundreds = 2 thousands and 7 hundreds. Write the 7 under hundreds' column, and add the 2 to thousands' column. The sum of the thousands, $2 + 1 + 3, = 6$, which write under thousands' column.

OPERATION

$$\begin{array}{r} 3415 \\ 503 \\ 1870 \\ \underline{922} \\ 6710 \end{array}$$

For convenience, the addition begins at the right-hand column, with the units of the lowest order, so that, if the sum of the figures in any column exceeds 9, the tens can be *carried* to the sum of the next higher order.

NOTE. — To illustrate the greater convenience of adding the units' column first, take the above example.

SOLUTION. — Commencing the addition with the thousands' column, the sum is 4; next adding the hundreds, the sum is 26 hundreds, which equal 2 thousands and 6 hundreds; next adding the tens, the sum is 10 tens, equal to 1 hundred; and finally adding the units, the sum is 10 units, equal to 1 ten. As these sums have also to be added, this much extra work must be done in order to complete the solution.

OPERATION

$$\begin{array}{l} 3415 \\ 503 \\ 1870 \\ \underline{922} \\ 4 \\ 26 \\ 10 \\ \underline{10} \\ 6710 \end{array}$$

20. Rule. — 1. *Write the numbers to be added, so that figures of the same order may stand in the same column.*

2. *Begin at the right hand, and add each column separately. Place the units obtained by adding each column under it, and carry the tens to the next higher order. Write down the entire sum of the last column.*

Proof. — *Add the columns downwards, commencing with he column of units. If the results agree, the work is probably correct.*

1. Find the sum of 3745, 2831, 5983, and 7665.

OPERATION

3745
2831
5983
7665
20224
321

In adding long columns of figures, it is convenient to retain the numbers carried. This may be done by placing them in smaller figures under their proper columns, as 3, 2, 1, in the margin.

Add:

2.	3.	4.	5.	6.	7.
184	204	103	495	384	1065
216	302	405	207	438	6317
135	401	764	185	348	5183
320	311	573	825	843	7102
413	109	127	403	483	3251
101	43	205	325	834	6044
1369	1370	2177	2440	3330	28962

8.	9.	10.	11.	12.
3725	5943	82703	987462	6840325
5834	6427	102	478345	7314268
4261	8204	6005	610628	3751954
7203	7336	759	423158	6287539
21023	27910	89569	2499593	24194086

13.	14.	15.	16.	17.
3686	9898	4356	893742	234567
4724	8989	6342	743698	765432
6583	4545	7989	437821	987654
5798	5454	4878	643567	456789
6953	6363	6749	892742	778899
27744	35249	30314	3611570	3223341

18.	19.	20.	21.
5493275	4819	18356	849627
6182463	9263	49276	532472
9538719	2752	94678	293784
2645834	8375	36525	468135
8256386	6498	42983	926547
32116677	31707	241818	3070565

22.	23.	24.	25.	26.
7421	6873	4729	237285	884261
6322	2196	6234	64371	724353
798	583	5781	2143	416213
4352	79	3143	842	598624
547	684	7182	55	784344
674	4348	6989	789	627517
2315	7896	7222	4621	843641
7218	233	6643	15115	47821
1847	594	7859	647890	52348
5721	6483	6742	77442	2932
6843	7542	8982	84931	4751
4722	3967	3451	894623	896
5976	29	8692	446217	722
6843	478	7341	134162	823344
1234	1717	6822	192317	874132
62833	43702	97812	2802803	6685899

27. $11 + 22 + 33 + 44 + 55 =$ how many? 165.

28. $23 + 41 + 74 + 83 + 16 =$ how many? 237.

29. $45 + 19 + 32 + 74 + 55 =$ how many? 225.

30. $51 + 48 + 76 + 85 + 4 =$ how many? 264.

31. $263 + 104 + 321 + 155 =$ how many? 843.

32. $94753 + 2847 + 93688 + 9386 + 258 + 3456$ are how many? 204388.

Write in columns and add:

33. 263, 104, 321, 155. 843.

34. 943, 756, 408, 696, 713, 559. 4075.

35. 615, 23, 14, 439, 98, 76. 1265.

36. 3421, 549, 314, 670, 2811, 1598. 9363.

37. 94753, 2847, 93688, 9386, 258, 3456. 204388.

38. 312742, 401009, 71268, 314, 39, 71620. 856992.

39. 2567, 16499, 888764, 41239, 5724, 8965. 963758.

40. 329876, 555555, 909090, 784378, 870960, 199999. 3649858.

41. 987654, 321234, 567897, 123456, 749947, 403209. 3153397.

42. 1065310, 72444, 3619, 412, 37, 92654. 1234476.

43. 4013009, 42645, 24, 132, 2460, 1709. 4059979.

44. January has 31 days; February, 28; March, 31; April, 30; and May, 31. How many days are there in these five months? 151.

45. June has 30 days; July, 31; August, 31; September, 30; October, 31. How many days are there in all? 153.

46. The first five months of a common year have 151 days, the next five have 153 days, November has 30, and December, 31. How many days are there in the whole year? 365.

47. I bought four pieces of muslin: the first contained 50 yards, the second, 65, the third, 42, and the fourth, 89. How many yards were there in all? 246 yd.

48. I owe one man \$245, another \$325, a third \$187, a fourth \$96. How much do I owe? \$853.

49. George Washington was born A.D. 1732, and lived 67 years. In what year did he die? 1799.

50. Alfred the Great died A.D. 901; thence to the signing of Magna Charta was 314 years; thence to the American Revolution, 560 years. In what year did the American Revolution begin? 1775.

51. A has four flocks of sheep: in the first there are 65 sheep and 43 lambs; in the second, 187 sheep and 105 lambs; in the third, 370 sheep and 243 lambs; in the fourth, 416 sheep and 95 lambs. How many sheep and lambs has he? 1038 sheep, and 486 lambs.

52. A man bought 15 barrels of pork for \$285, 9 barrels for \$144, 13 barrels for \$234, and 24 barrels for \$408. How many barrels did he buy, and how many dollars did he pay? 61 barrels, and \$1071.

53. The first of four numbers is 287; the second, 596; the third, 841; and the fourth, as much as the first three. What is their sum? 3448.

54. Some of the Pyramids of Egypt were built 1700 years before the founding of Carthage; Carthage was founded 47 years before Rome. Rome was founded 753 years before the Christian era. How many years before Christ were these Pyramids built? 2500.

55. Add three thousand five; forty-two thousand six hundred twenty-seven; one hundred five; three hundred seven thousand four; eighty thousand seventy-nine; three hundred twenty thousand six hundred. 753420.

56. Add two hundred seventy-five thousand four hundred thirty-two; four hundred two thousand thirty; three hundred thousand five; eight hundred seventy-two thousand twenty-six; four million two thousand three hundred forty-seven. 5851840.

57. At the battle of Waterloo, which took place June 18, 1815, the estimated loss of the English was 12000; of the French, 40000; of the Prussians, 38000; of the Belgians, 8000; of the Hanoverians, 3500. What was the total loss of life in this battle? 101500.

58. Add eight hundred eighty million eight hundred eighty-nine; two million two thousand two; seventy-seven million four hundred thirty-six thousand; two hundred six million five thousand two hundred seven; forty-nine thousand three; nine hundred ninety million nineteen thousand nine hundred nineteen. 2155513020.

59. North America has an area of 9268837 square miles; South America, 6887794 square miles; and the West Indies, 94398 square miles. What is the area of the entire continent? 16251029 sq. mi.

60. A man pays $600 for a lot, $1325 for building materials, $30 for digging the cellar, $120 for stonework, $250 for brickwork, $140 for carpenter-work, $120 for plastering, and $115 for painting. How much does his house and lot cost him? $2700.

61. A man bequeaths $7850 to his wife, $3275 to each of his two sons, and $2650 to each of his three daughters. What is the amount of his bequest? $22350.

62. A merchant spent \$8785 for dress goods, and \$12789 for sheetings. He sold the dress goods at a profit of \$878, and the sheetings at a profit of \$1250. For how much did he sell the whole? \$23702.

63. A merchant began business with \$7000 cash, goods worth \$12875, bank stock worth \$5600, and other stocks worth \$4785. In one year he gained \$3500. How much was he worth at the end of the year? \$33760.

64. A house has two parlors, each requiring 30 yards of carpet; four bedrooms, each requiring 25 yards; a dining room and sitting room, each requiring 20 yards. How many yards are required to carpet the entire house? 200 yd.

65. The population of a certain city was 33587 in 1900. In 1901 it had increased by 1245; and in 1902 by 2067 more. What was the population in 1902? 36899.

66. The village of Clayton is 463 miles east of Mayville, and Bartow is 756 miles west of Mayville. How far is it from Clayton to Bartow? 1219 mi.

67. The area of Maine is 33040 square miles; of New Hampshire, 9305 square miles; of Vermont, 9565 square miles; of Massachusetts, 8315 square miles; of Rhode Island, 1250 square miles; of Connecticut, 4990 square miles. What is the total area of New England?

66465 square miles.

68. The area of the United States up to 1897 was 3681661 square miles. Since then there have been added the territory of Hawaii containing 6449 square miles; Porto Rico, 3531 square miles; Philippine Islands, 114410 square miles; Guam, 150 square miles; Tutuila, 77 square miles; and Wake Island, 1 square mile. What is the present area of the United States? 3806279 square miles.

SUBTRACTION

21. 1. If you have 9 apples, and give 4 away, how many will you have left? *Ans.* 5 apples.

Why? Because 4 apples from 9 apples are 5 apples.

2. Frank had 15 cents. After spending 7, how many were left? *Ans.* 8 cents. Why?

3. If you take 8 from 13, how many are left? *Ans.* 5.

4. If I have 25 cents, and spend 10 cents for a lead pencil, how much will I have left? *Ans.* 15 cents.

5. Twelve from twenty leaves how many? *Ans.* 8.

22. The operation in the preceding examples is termed *subtraction.*

Subtraction is the process of taking one number from another or of finding the difference between two numbers.

The number from which another is to be subtracted is called the **minuend**; the number to be subtracted the **subtrahend**; and the number left after subtraction, the **difference** or **remainder**.

23. The **sign of subtraction** ($-$) is called *minus*, meaning *less*. When placed between two numbers, it denotes that the number on the right is to be taken from the one on the left. Thus, $8 - 5 = 3$ means that 5 is to be taken from 8, and is read, 8 *minus* 5 *equals* 3.

Subtraction Table

$2-2=0$	$3-3=0$	$4-4=0$	$5-5=0$
$3-2=1$	$4-3=1$	$5-4=1$	$6-5=1$
$4-2=2$	$5-3=2$	$6-4=2$	$7-5=2$
$5-2=3$	$6-3=3$	$7-4=3$	$8-5=3$
$6-2=4$	$7-3=4$	$8-4=4$	$9-5=4$
$7-2=5$	$8-3=5$	$9-4=5$	$10-5=5$
$8-2=6$	$9-3=6$	$10-4=6$	$11-5=6$
$9-2=7$	$10-3=7$	$11-4=7$	$12-5=7$
$10-2=8$	$11-3=8$	$12-4=8$	$13-5=8$
$11-2=9$	$12-3=9$	$13-4=9$	$14-5=9$
$6-6=0$	$7-7=0$	$8-8=0$	$9-9=0$
$7-6=1$	$8-7=1$	$9-8=1$	$10-9=1$
$8-6=2$	$9-7=2$	$10-8=2$	$11-9=2$
$9-6=3$	$10-7=3$	$11-8=3$	$12-9=3$
$10-6=4$	$11-7=4$	$12-8=4$	$13-9=4$
$11-6=5$	$12-7=5$	$13-8=5$	$14-9=5$
$12-6=6$	$13-7=6$	$14-8=6$	$15-9=6$
$13-6=7$	$14-7=7$	$15-8=7$	$16-9=7$
$14-6=8$	$15-7=8$	$16-8=8$	$17-9=8$
$15-6=9$	$16-7=9$	$17-8=9$	$18-9=9$

24. When each figure of the subtrahend is not greater than the corresponding figure of the minuend:

1. A man having $135, spent $112. How much had he left?

Solution. — For convenience, write the smaller number under the greater, with units of the *same order* in the same column.

Begin at the right; 2 from 5 leaves 3, which put in units' place; 1 from 3 leaves 2, which put in tens' place; 1 from 1 leaves 0, and, as there are no figures on the left of this, the place is vacant.

OPERATION

135, minuend.
112, subtrahend.
23, remainder.

2. A farmer having 245 sheep, sold 123. How many sheep had he left? 122.

3. A man bought a farm for $751, and sold it for $875. How much did he gain? $124.

Find the difference between:

4. 734 and 531. 203.
5. 8752 and 3421. 5331.
6. 79484 and 25163. 54321.
7. 49528 and 16415. 33113.

25. When the lower figure in any order is greater than the upper:

1. James had 13 cents. After spending 5, how many cents had he left?

OPERATION

$$\begin{array}{r} 13 \\ \underline{5} \\ 8 \end{array}$$

SOLUTION.—5 cannot be subtracted from 3, but it can be subtracted from 13; 5 from 13 leaves 8.

2. From 73 subtract 45.

OPERATION

$$\begin{array}{r} 73 \\ \underline{45} \\ 28 \end{array}$$

SOLUTION.—5 units cannot be taken from 3 units. Therefore change 1 of the 7 tens to 10 units and add these to the 3 units, making 13 units; then, subtract the 5 units, and there will remain 8 units, to be put in units' place. Since 1 ten is taken from the 7 tens, there remain but 6 tens. Subtract 4 tens from 6 tens and put the remainder, 2 tens, in tens' place. The difference is 28.

NOTES.—1. Instead of actually taking 1 ten from the 7 tens, and adding it to the 3 units, the operation is performed *mentally;* thus, 5 from 13 leaves 8, and 4 from 6 leaves 2.

2. In such cases the value of the upper number is not changed, since the 1 ten which is taken from the order of tens is added to the number in the order of units.

3. After increasing the units by 10, instead of considering the next figure of the upper number as *diminished* by 1, the result will be the same, if the next figure of the lower number is *increased* by 1; thus, in the previous example, instead of diminishing the 7 tens by 1,

add 1 to the 4 tens, which makes 5; thus, 5 from 13 leaves 8, and 5 from 7 leaves 2. This process depends upon the principle that adding a number to the subtrahend gives the same result as subtracting it from the minuend. Since the subtrahend is to be subtracted from the minuend whatever is *added to* the subtrahend will be *subtracted from* the minuend.

3. Find the difference between 805 and 637.

OPERATION
805
637
168

SOLUTION.—Writing the smaller number under the greater, with units of the same order in the same column, it is required to subtract the 7 units from 5 units.

7 units cannot be taken from 5 units. As there are no tens we change 1 of the 8 hundreds to 10 tens, leaving 7 in hundreds' place. We next change 1 of these 10 tens to units, leaving 9 tens and 15 units.

Subtracting 7 from 15, 8 units are left, to be written in units' place; next, subtracting 3 tens from 9 tens, there are left 6 tens, to be written in tens' place; lastly, subtracting 6 hundreds from 7 hundreds, there remains 1 hundred, to be written in hundreds' place.

NOTES.—1. If the 5 units are increased by 10, say 7 from 15 leaves 8; then, increasing the 3 by 1, say 4 from 10 leaves 6; then, increasing 6 by 1, say 7 from 8 leaves 1.

2. Begin at the right to subtract, so that if any lower figure is greater than the upper, 1 may be borrowed from a higher order.

3. If the difference of two numbers is added to the smaller number, the sum will be equal to the greater. Thus, if 5 subtracted from 8 leaves 3, then 3 added to 5 will equal 8.

Rule.—1. *Write the smaller number under the greater, placing figures of the same order in the same column.*

2. *Beginning at the right hand, subtract each figure from the one directly over it, and write the remainder beneath.*

3. *If the lower figure exceeds the upper, add ten to the upper figure, and subtract the lower from it. Then take one from the next upper figure, or add one to the next lower figure, and subtract as before.*

Proof. — *Add the remainder to the subtrahend. If the sum is equal to the minuend, the work is correct.*

	1.	2.	3.	4.
Minuends,	7640	860012	4500120	3860000
Subtrahends,	1234	430021	2910221	120901
Remainders,	6406	429991	1589899	3739099
Proof,	7640	860012	4500120	3860000

Subtract:

5.	1234567 from 4444444.	3209877.
6.	15161718 from 91516171.	76354453.
7.	34992884 from 63046571.	28053687.
8.	19019 from 20010.	991.
9.	405022 from 2000687.	1595665.
10.	1009006 from 2020930.	1011924.
11.	50082 from 912010.	861928.
12.	9238715 from 18126402.	8887687.
13.	9909090009 from 10900900900.	991810891.
14.	4004 from 4000000.	3995996.
15.	53845248 from 153425178.	99579930.
16.	10001001 from 100000000.	89998999.
17.	13201 from 17102102.	17088901.

Subtract and prove:

18.	876 − 385.	491.	23.	540 − 398.	142.
19.	105 − 96.	9.	24.	209 − 136.	73.
20.	908 − 399.	509.	25.	313 − 267.	46.
21.	765 − 576.	189.	26.	500 − 347.	153.
22.	620 − 434.	186.	27.	490 − 285.	205.

28. 4187 − 3098. 1089.

29. 5824 − 4778. 1046.

30. 8932 − 6596. 2336.

31. 3795 − 2499. 1296.

32. 6349 − 5303. 1046.

33. 1471 − 1284. 187.

34. 2563 − 2155. 408.

35. 7656 − 7577. 79.

36. 9208 − 8469. 739.

37. 5090 − 4345. 745.

38. 29432 − 13278. 16154.

39. 38416 − 29547. 8869.

40. 47000 − 8755. 38245.

41. 56559 − 966. 55593.

42. 65000 − 2044. 62956.

43. 74680 − 9933. 64747.

44. 83700 − 992. 82708.

45. 92000 − 88091. 3909.

46. 11300 − 10949. 351.

47. 80200 − 79858. 342.

48. Take 17 cents from 63 cents. 46 cents.

49. A carriage cost $137, and a horse $65. How much more than the horse did the carriage cost? $72.

50. A tree 75 feet high was broken; the part that fell was 37 feet long. How high was the stump? 38 ft.

51. America was discovered by Columbus in 1492. How many years had elapsed in 1900? 408.

52. I deposited in the bank $1840, and drew out $475. How many dollars had I left? $1365.

53. A man has property worth $10104, and owes debts to the amount of $7426. When his debts are paid, how much will be left? $2678.

54. A man having $100000, gave away $11. How much had he left? $99989.

55. What is the difference in height between the volcano of Mt. Pelée, 4430 ft. high, and that of Mt. Vesuvius, 4205 ft. high? 225 ft.

56. By how much does the area of Lake Superior, which is 31200 square miles, exceed that of Lake Ontario, which is 7240 square miles? 23960 sq. mi.

57. The minuend is 15649, which is 2408 more than the subtrahend. What is the subtrahend? 13241.

58. In 1900 the population of New York City was 3437202, and that of Chicago 1698575. How many more people were there in New York than in Chicago? 1738627.

59. How far apart are two cities, one of which is 2346 miles from Boston, and the other 4335 miles in the same direction from Boston? 1989 mi.

60. A man invested $30000 in business; at the end of the first year all his assets amounted to only $26967. How much had he lost? $3033.

61. Mr. Beard has $297420 less than Mr. Hall, who has $456721. How much has Mr. Beard? $159301.

62. If I borrow $32450 and pay back $19275, how much do I still owe? $13175.

63. The sum of two numbers is 619346, and the greater is 452020. What is the less? 167326.

64. The 250th anniversary of the founding of a society was celebrated in 1903. When was the society founded? 1653.

65. Previous to a fire the contents of a store was valued at $25690; after the fire at $18040. How much was lost by the fire? $7650.

Examples in Addition and Subtraction

26. 1. $275 + 381 + 625 - 1098 = ?$ 183.

2. $6723 - 479 - 347 - 228 = ?$ 5669.

3. I owe three notes, whose sum is $1300, one note being for $250, and another for $650. What is the amount of the third note? $400.

4. In January, 1903, a merchant bought goods to the amount of $2675; in February, $4375; and in March, $1897. After making one payment of $3000, and another of $4947, how much did he still owe? $1000.

5. Mr. Jones deposited in a bank on Monday $450; on Tuesday, $725; on Wednesday, $1235; on Thursday, $4675; and on Friday, $1727. On Saturday morning he drew out $5935, and Saturday afternoon, $877. How much money had he left in bank? $2000.

6. At the end of one year I found I had spent $2300. Of this amount, $350 was paid for board, $125 for clothing, $375 for books, $150 for incidentals, and the remainder for two acres of ground. How much did the two acres cost? $1300.

7. A speculator bought three houses. For the first he paid $4875; for the second, $2250 more than for the first; and for the third, $3725. He afterward sold them all for $20,838. How much did he gain? $5113.

8. A man owns property valued at $49570, of which $16785 are in personal property, and $24937 in real estate; the remainder is deposited in bank. How much has he in bank? $7848.

9. A merchant bought a bill of goods for $7895, and paid $175 for freight, and $3 for cartage. He sold the goods for $10093. How much did he gain? $2020.

10. The following is Mr. Brown's private account for two weeks: First week, received $50 for salary, and spent $25 for clothing, $8 for board, $1 for washing, and $5 for sundries. Second week, received $50 for salary, loaned $35 to Tom Jones, paid $8 for board, $1 for washing, and $8 for sundries. How much did Mr. Brown have at the end of the two weeks? $9.

11. A farmer invested \$10000, as follows: in land, \$5750; in horses, \$925; in cattle, \$1575; in hogs, \$675; and the remainder in implements and tools. How much did he invest in implements and tools? \$1075.

12. A speculator on Monday gained \$4625; on Tuesday, \$3785; on Wednesday he lost \$6955; on Thursday he lost \$895; on Friday he gained \$985; and on Saturday he lost \$1375. How much did he gain during the entire week? \$170.

13. A farmer raised 2040 bushels of rye and 1059 bushels of wheat. After selling 1050 bushels of rye and 985 bushels of wheat, how many bushels of both together had he left? 1064 bu.

14. There are 21360 newspapers printed in the United States and Canada, 8000 in Great Britain, 6000 in Germany, and 4300 in France. How many more are printed in the United States and Canada than in Great Britain, Germany, and France together? 3060.

15. If 111100000 people in the world speak English, 51200000 French, and 42800000 Spanish, how many more people speak English than French and Spanish together? 17100000.

16. The area of North America is 9363235 square miles; of South America, 6887794 square miles; of Europe, 3857504 square miles. How much greater is the area of North and South America together than that of Europe? 12393525 sq. mi.

17. Congress voted \$200000 for the relief of the victims of the volcanic eruption in the island of Martinique. If \$110075 were spent for food and \$46025 for clothing, how much remained for other expenses? \$43900.

MULTIPLICATION

27. **1.** If one orange costs 2 cents, how much will 3 oranges cost?

SOLUTION. — Three oranges will cost 3 times as much as one orange; that is, 2 cents taken 3 times: $2 + 2 + 2 = 6$.

2. If one lemon costs 3 cents, how many cents will 4 lemons cost? *Ans.* $3 + 3 + 3 + 3 = 12$.

3. At 4 cents apiece, how much will 6 pencils cost? *Ans.* $4 + 4 + 4 + 4 + 4 + 4 = 24$.

4. In an orchard there are 4 rows of trees, and in each row, 21 trees. How many trees are there in the orchard?

SOLUTION. — 1. By writing 21 four times, as in the margin, and adding, the whole number of trees is 84.

2. Instead of writing 21 four times, write it once, place the number 4 underneath, to indicate the number of times 21 is to be taken, and say, 4 times 1 are 4, which put in units' place; then, 4 times 2 are 8, which put in tens' place; the result is 84, the same as found by addition.

1st row, 21 trees.
2d row, 21 trees.
3d row, 21 trees.
4th row, 21 trees.

$$84$$

OPERATION.

$$\begin{array}{r} 21 \\ \underline{4} \\ 84 \end{array}$$

The latter method of obtaining the result is called *multiplication*. Therefore,

Multiplication is a short method of performing many additions of the same number.

28. **Multiplication** is the process of taking one number as many times as there are units in another.

The **multiplicand** is the number to be taken or multiplied; the **multiplier** is the number denoting how many times the multiplicand is taken; the result is termed the **product**.

Thus, 4 times 5 are 20; 5 is the *multiplicand*, 4 the *multiplier*, and 20 the *product*.

The multiplicand and multiplier are together called **factors**, because they make or produce the product.

29. The **sign of multiplication** is $\times$. It is read *multiplied by* when the multiplicand precedes it, and *times* when the multiplier precedes it.

Thus, 8 dollars $\times$ 5 is read 8 dollars multiplied by 5; 5 $\times$ 8 dollars is read 5 times 8 dollars; 6 $\times$ 4 is read 6 multiplied by 4 when 6 is the multiplicand, and 6 times 4 when 6 is the multiplier.

In the table, read the sign $\times$ as *times;* thus, 2 times 2 are 4; 2 times 3 are 6.

MULTIPLICATION TABLE

$1 \times 1 = 1$	$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$
$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$
$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$
$4 \times 1 = 4$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$
$5 \times 1 = 5$	$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$
$6 \times 1 = 6$	$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 4 = 24$
$7 \times 1 = 7$	$7 \times 2 = 14$	$7 \times 3 = 21$	$7 \times 4 = 28$
$8 \times 1 = 8$	$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 4 = 32$
$9 \times 1 = 9$	$9 \times 2 = 18$	$9 \times 3 = 27$	$9 \times 4 = 36$
$10 \times 1 = 10$	$10 \times 2 = 20$	$10 \times 3 = 30$	$10 \times 4 = 40$
$11 \times 1 = 11$	$11 \times 2 = 22$	$11 \times 3 = 33$	$11 \times 4 = 44$
$12 \times 1 = 12$	$12 \times 2 = 24$	$12 \times 3 = 36$	$12 \times 4 = 48$

$1 \times 5 = 5$	$1 \times 6 = 6$	$1 \times 7 = 7$	$1 \times 8 = 8$
$2 \times 5 = 10$	$2 \times 6 = 12$	$2 \times 7 = 14$	$2 \times 8 = 16$
$3 \times 5 = 15$	$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$
$4 \times 5 = 20$	$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$
$5 \times 5 = 25$	$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$
$6 \times 5 = 30$	$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$
$7 \times 5 = 35$	$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$
$8 \times 5 = 40$	$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$
$9 \times 5 = 45$	$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$
$10 \times 5 = 50$	$10 \times 6 = 60$	$10 \times 7 = 70$	$10 \times 8 = 80$
$11 \times 5 = 55$	$11 \times 6 = 66$	$11 \times 7 = 77$	$11 \times 8 = 88$
$12 \times 5 = 60$	$12 \times 6 = 72$	$12 \times 7 = 84$	$12 \times 8 = 96$
$1 \times 9 = 9$	$1 \times 10 = 10$	$1 \times 11 = 11$	$1 \times 12 = 12$
$2 \times 9 = 18$	$2 \times 10 = 20$	$2 \times 11 = 22$	$2 \times 12 = 24$
$3 \times 9 = 27$	$3 \times 10 = 30$	$3 \times 11 = 33$	$3 \times 12 = 36$
$4 \times 9 = 36$	$4 \times 10 = 40$	$4 \times 11 = 44$	$4 \times 12 = 48$
$5 \times 9 = 45$	$5 \times 10 = 50$	$5 \times 11 = 55$	$5 \times 12 = 60$
$6 \times 9 = 54$	$6 \times 10 = 60$	$6 \times 11 = 66$	$6 \times 12 = 72$
$7 \times 9 = 63$	$7 \times 10 = 70$	$7 \times 11 = 77$	$7 \times 12 = 84$
$8 \times 9 = 72$	$8 \times 10 = 80$	$8 \times 11 = 88$	$8 \times 12 = 96$
$9 \times 9 = 81$	$9 \times 10 = 90$	$9 \times 11 = 99$	$9 \times 12 = 108$
$10 \times 9 = 90$	$10 \times 10 = 100$	$10 \times 11 = 110$	$10 \times 12 = 120$
$11 \times 9 = 99$	$11 \times 10 = 110$	$11 \times 11 = 121$	$11 \times 12 = 132$
$12 \times 9 = 108$	$12 \times 10 = 120$	$12 \times 11 = 132$	$12 \times 12 = 144$

30. The product of two numbers is not altered by changing the order of the factors. Thus, $6 \times 4 = 24$, and $4 \times 6 = 24$.

A number used without reference to any particular thing is called an **abstract number**; as, 2, 5, 8.

A number used in connection with some thing is called **a concrete number**; as, 2 chairs, 5 dollars, 8 pens.

NOTE. — All *numbers* are in themselves abstract whether the kind of thing numbered is mentioned or not, but the distinction between abstract and concrete numbers as above defined is often convenient.

The multiplier is always regarded as an **abstract number**, and the product is always of the same denomination as the multiplicand. Thus, in the example $5 \times 3 = 15$, the multiplicand is an abstract number; in the example 5×3 cents $= 15$ cents, the multiplicand is a concrete number.

Either factor may be used as multiplier when the numbers are abstract. In practice the smaller number is generally used for convenience.

31. I. When the multiplier does not exceed 12:

1. How many yards of cloth are there in 3 pieces, each containing 123 yards?

SOLUTION. — Place the multiplier under the multiplicand; 3 times 3 are 9 (units); write 9 in units' place; 3 times 2 are 6 (tens); write 6 in tens' place; 3 times 1 are 3 (hundreds); write 3 in hundreds' place.

OPERATION

123,	multiplicand.
3,	multiplier.
369,	product.

2. How much will 2 pianos cost at $231 each? $462.

3. How much will 3 horses cost at $132 each? $396.

4. What is the product of 4×201? 804.

5. What is the product of 3×2301? 6903.

6. At $43 an acre, how much will 5 acres of land cost?

SOLUTION. — 5 times 3 are 15 (units); write the 5 in units' place, and reserve the 1 (ten) to add to the tens; 5 times 4 are 20, and 1 reserved are 21 (tens); write 1 in tens' place and 2 in hundreds' place.

OPERATION

$43
5
$215

Rule. — 1. *Write the multiplicand, with the multiplier under it, and draw a line beneath.*

2. *Begin with units; Multiply each figure of the multiplicand by the multiplier, adding the tens to the tens' column as in addition.*

NOTE.— Begin at the right hand to multiply, for convenience, so that the excess of tens in any lower order may be added to the order next higher.

	7.	8.	9.	10.
Multiplicand,	5142	4184	3172	41834
Multiplier,	5	6	5	7
Product,	25710	25104	15860	292838

Multiply:

11.	49 by 3.	147.
12.	57 by 4.	228.
13.	128 by 5.	640.
14.	367 by 6.	2202.
15.	1427 by 7.	9989.
16.	19645 by 8.	157160.
17.	44386 by 9.	399474.
18.	708324 by 7.	4958268.
19.	96432 by 10.	964320.
20.	46782 by 11.	514602.
21.	86458 by 12.	1037496.

II. When the multiplier exceeds 12:

22. What is 25 times 43?

ANALYSIS.— Since 25 is equal to 2 tens and 5 units, that is, 20 + 5, multiply by 5 and write the product, 215; then multiply by the 2 tens, and write the product, 8 hundreds and 6 tens, under the 2 hundreds and 1 ten.

Multiplying by 5 units gives 5 times 43, and multiplying by 2 tens gives 20 times 43; add them, because 5 times 43 and 20 times 43 equal 25 times 43.

OPERATION

$$\begin{array}{rl} 43 & \\ \underline{25} & \\ 215 & = 5 \times 43 \\ \underline{86} & = 20 \times 43 \\ 1075 & = 25 \times 43 \end{array}$$

Hence, multiply by the units' figure of the multiplier, and write the product so that the right-hand figure will fall in

units' place; then multiply by the tens' figure, and write the right-hand figure of the product in the tens' place.

Therefore, in multiplying by a figure of any order, write the right-hand figure of the product in the same order as the multiplier.

NOTE. — The products of the multiplicand by the separate figures of the multiplier are called **partial products**.

General Rule. — 1. *Write the multiplier under the multiplicand, placing figures of the same order in a column.*

2. *Multiply the multiplicand by each figure of the multiplier in succession, beginning with units, always setting the right-hand figure of each product under that figure of the multiplier which produces it.*

3. *Add the partial products; their sum will be the product sought.*

Proof. — *Multiply the multiplier by the multiplicand. If the product thus obtained is the same as the first product, the work is probably correct.*

23. Multiply 2345 by 123

SOLUTION

2345, multiplicand.	
123, multiplier.	
7035	$= 3 \times 2345$
469	$= 20 \times 2345$
2345	$= 100 \times 2345$
288435	$= 123 \times 2345$

PROOF

123, multiplier.	
2345, multiplicand.	
615	$= 5 \times 123$
492	$= 40 \times 123$
369	$= 300 \times 123$
246	$= 2000 \times 123$
288435	$= 2345 \times 123$

24. Multiply 327 by 203.

REMARK. — When there is a cipher in the multiplier, multiply only by the other figures, being careful to place the right-hand figure of each partial product under the multiplying figure.

OPERATION

```
  327
  203
  981
654
66381
```

25. $13 \times 285 = 3055.$
26. $19 \times 346 = 6574.$
27. $29 \times 425 = 12325.$
28. $34 \times 518 = 17612.$
29. $37 \times 279 = 10323.$
30. $49 \times 869 = 42581.$
31. $57 \times 294 = 16758.$
32. $62 \times 429 = 26598.$
33. $76 \times 485 = 36860.$
34. $85 \times 624 = 53040.$
35. $97 \times 976 = 94672.$
36. $364 \times 342 = 124488.$
37. $526 \times 376 = 197776.$
38. $536 \times 476 = 255136.$
39. $215 \times 2187 = 470205.$
40. $276 \times 3489 = 962964.$
41. $365 \times 1646 = 600790.$
42. $635 \times 8432 = 5354320.$

Multiply:

43. 6874 by 829. 5698546.
44. 2873 by 1823. 5237479.
45. 4786 by 3497. 16736642.
46. 87603 by 9865. 864203595.
47. 83457 by 6835. 570428595.
48. 31624 by 7138. 225732112.

49. How much will 189 barrels of flour cost, at \$4 a barrel? \$756.

50. How much will 823 office desks cost, at \$12 apiece? \$9876.

51. How much will 675 pounds of cheese cost, at 13 cents a pound? 8775 cents.

52. How much will 248 bushels of oats cost, at 48 cents a bushel? 11904 cents.

53. If a man travels 28 miles a day, how many miles will he travel in 152 days? 4256 miles.

54. There are 1760 yards in one mile. How many yards are there in 209 miles? 367840 yards.

55. There are 24 hours in a day, and 365 days in a year. If a ship sails 8 miles an hour, how far will she sail in a year? 70080 miles.

56. Multiply two thousand twenty-nine by one thousand seven. 2043203.

57. Multiply eighty thousand four hundred one by sixty thousand seven. 4824622807.

58. Multiply one hundred one thousand thirty-two by twenty thousand one. 2020741032.

59. A grocer bought 860 pounds of sugar for 4 cents a pound. How much did he pay for the sugar? 3440 cents.

60. A grocer bought 36 gallons of molasses for 45 cents a gallon, and sold it for 55 cents a gallon. How much did he gain? 360 cents.

61. A commission merchant sold 2650 bushels of wheat for a farmer, at 95 cents a bushel, and charged him 2 cents a bushel for selling. How much money was due the farmer? 246450 cents.

62. A farmer bought 6 horses of one man for 75 dollars each, and 5 horses of another for 125 dollars each, and sold them all for 150 dollars each. How many dollars did he gain? $575.

63. A merchant bought one box of goods for 250 dollars, two more for 325 dollars each, and three more for 175 dollars each. He sold them all so as to gain 356 dollars. For how much did he sell them? $1781.

64. A farmer bought 24 sheep, at 5 dollars a head; 36 hogs, at 14 dollars a head; and 9 cows, at 45 dollars a head. When he sold them all, he lost 275 dollars. For how much did he sell them? $754.

65. To 75×37 add 85×54, and subtract 5284. 2081.

66. To 69×53 add 48×27, and subtract 4279. 674.

67. A man bought 85 bags of coffee, of 63 pounds each, at 20 cents a pound. How much did it cost? 107100 cents.

68. If a man takes 2235 steps in walking a mile, how many steps will he take in walking 28 miles? 62580 steps.

69. Two steamers start from the same place and sail in opposite directions. The first sails 16 miles an hour and the second 17 miles an hour. How far apart will they be in 2 days of 24 hours each? 1584 miles.

70. If 125 tons of iron rail are used for a mile of railroad, how many tons will be used for 248 miles? 31,000 tons.

71. A barrel of flour weighs 196 pounds. How many pounds will 2347 barrels weigh? 460012 pounds.

72. A freight train consisting of 18 cars is laden with flour. Each car contains 75 barrels of flour of 196 pounds each. How many pounds of flour are in the train?

264600 pounds.

CONTRACTIONS IN MULTIPLICATION

32. When the multiplier can be separated into factors:

1. How much will 15 melons cost, at 8 cents each?

OPERATION

Cost of	1 melon,	8¢
		5
Cost of	5 melons,	40¢
		3
Cost of	15 melons,	120¢

Analysis. — Since 15 is 3 times 5, 15 melons will cost 3 times as much as 5 melons.

Therefore, instead of multiplying 8 by 15, first find the cost of 5 melons, by multiplying 8 cents by 5; then take 3 times that product for the cost of 15 melons.

Rule. — 1. *Separate the multiplier into two or more factors.*

2. *Multiply the multiplicand by one of the factors, and this product by another factor, till every factor is used; the last product will be the one required.*

NOTE.—Do not confound the *factors* of a number with the *parts* into which it may be separated. Thus, the factors of 15 are 5 and 3, while the parts into which 15 may be separated are any numbers whose *sum* equals 15; as, 7 and 8; or 2, 9, and 4.

2. How much will 24 acres of land cost at $124 an acre? $2976.

3. How far will a ship sail in 56 weeks, at the rate of 1512 miles per week? 84672 miles.

4. How many pounds of iron are there in 54 loads, each weighing 2873 pounds? 155142 pounds.

5. Multiply 2874 by 72. 206928.

6. Multiply 8074 by 108. 871992.

33. When the multiplier is 1 with ciphers annexed; as 10, 100, 1000, etc.:

1. Placing *one* cipher on the right of a number (§ 8) changes the units into tens, the tens into hundreds, and so on, and therefore, *multiplies the number by ten;* thus, annex one cipher to 25, and it becomes 250.

2. Annexing *two* ciphers changes units into hundreds, tens into thousands, etc., and *multiplies the number by one hundred;* thus, annex two ciphers to 25, and it becomes 2500.

Rule.—*Annex as many ciphers to the multiplicand as there are ciphers in the multiplier, and the number thus formed will be the product required.*

Multiply:

1. 245 by 100. 24500.
2. 138 by 1000. 138000.
3. 428 by 10000. 4280000.
4. 872 by 100000. 87200000.
5. 9642 by 1000000. 9642000000.
6. 10045 by 1000000. 10045000000.

34. When there are ciphers at the right of one or both of the factors:

1. Find the product of 625 by 500.

OPERATION
625
500
312500

ANALYSIS.—The multiplier may be considered as composed of two factors: 5 and 100. Multiplying by 5, the product is 3125; the product of this number by 100 is 312500, which is the same as annexing two ciphers to the first product.

2. Find the product of 2300×170.

OPERATION
2300
170
161
23
391000

ANALYSIS.—The number 2300 may be regarded as composed of the two factors 23 and 100; and 170, of the two factors 17 and 10.

The product of 2300 by 170 is found by multiplying 23 by 17, and this product by 100, and the resulting product by 10 (§ 33); that is, by finding the product of 23 by 17, and then annexing 3 ciphers to the product, as there are 3 ciphers at the right of both factors.

Rule.—*Multiply without regarding the ciphers on the right of the factors; then annex to the product as many ciphers as are at the right of both factors.*

Multiply:

3.	2350 by 60.	141000.
4.	80300 by 450.	36135000.
5.	10240 by 3200.	32768000.
6.	9600 by 2400.	23040000.
7.	18001 by 26000.	468026000.
8.	8602 by 1030.	8860060.
9.	3007 by 9100.	27363700.
10.	80600 by 7002.	564361200.
11.	70302 by 80300.	5645250600.
12.	904000 by 10200.	9220800000.
13.	80360 by 25000.	2009000000.

DIVISION

35. 1. If you divide 6 apples into groups of 2 apples each, how many groups will there be?

Ans. 2 apples are contained 3 times in 6 apples; therefore, there will be 3 groups.

2. If you divide 6 apples equally between 2 boys, how many will each boy have?

Ans. Each boy will have one half of 6 apples, or 6 apples divided by 2, which are 3 apples.

How many times is 2 contained in 6? *Ans.* 3.
Why? Because 3 times 2 are 6.

What is one half of 6? *Ans.* 3.
Why? Because 3 multiplied by 2 = 6.

3. If you divide 8 peaches equally between 2 boys, how many will each have? *Ans.* 4 peaches. Why?

4. How many times is 2 contained in 10? What is one half of 10? *Ans.* 5. Why?

The process by which the preceding examples are solved is called *division*.

36. Division is the process of finding how many times one number is contained in another. It is also the process of separating a number into equal parts.

The **divisor** is the number by which to divide; the **dividend** is the number to be divided; the **quotient** is the

result obtained by division. The quotient shows how many times the divisor is contained in the dividend.

Thus, 3 is contained in 12, 4 times; here, 3 is the *divisor*, 12 the *dividend*, and 4 the *quotient*.

Since 3 is contained in 12 four times, 4 times 3 are 12; that is, the divisor and quotient multiplied together produce the dividend.

Since 3 and 4 are factors of the product 12, the divisor and quotient correspond to the factors in multiplication; the dividend, to the product. Therefore,

Division is the process of finding one of the factors of a product, when the other factor is known.

37. A boy has 8 cents. How many lemons can he buy, at 2 cents each?

ANALYSIS. — He can buy 4, because 4 lemons, at 2 cents each, will cost 8 cents.

The boy would give 2 cents for 1 lemon, and then have 6 cents left.

After giving 2 cents for the 2d lemon, he would have 4 cents left.

Then, giving 2 cents for the 3d, he would have 2 cents left.

Lastly, after giving 2 cents for the 4th, he would have nothing left.

	8 cents.
1st lemon,	2 cents.
Left,	6 cents.
2d lemon,	2 cents.
Left,	4 cents.
3d lemon,	2 cents.
Left,	2 cents.
4th lemon,	2 cents.
Left,	0 cents.

A natural method of performing this operation is by subtraction; but, when it is known *how many times* 2 can be subtracted from 8, instead of subtracting 2 four times, say 2 is contained in 8 four times, and 4 times 2 are 8. Therefore,

Division is a short method of making many subtractions of the same number.

The divisor is the number subtracted; the dividend is the number from which the subtraction has been made; the quotient shows how many subtractions have been made.

38. Division is indicated in three ways:

3)12, $\frac{12}{3}$, 12 ÷ 3, each means that 12 is to be divided by 3.

In using the first sign when the divisor does not exceed 12, draw a line under the dividend, and write the quotient beneath; if the divisor exceeds 12, draw a curved line on the right of the dividend, and place the quotient on the right of this.

The sign (÷) is read *divided by.*

$\begin{array}{r} 2\underline{)8} \\ 4 \end{array}$	$\begin{array}{l} 15)45(3 \\ \underline{45} \end{array}$	$\frac{15}{5} = 3$	$21 \div 3 = 7$

Division Table

1 ÷ 1 = 1	2 ÷ 2 = 1	3 ÷ 3 = 1	4 ÷ 4 = 1
2 ÷ 1 = 2	4 ÷ 2 = 2	6 ÷ 3 = 2	8 ÷ 4 = 2
3 ÷ 1 = 3	6 ÷ 2 = 3	9 ÷ 3 = 3	12 ÷ 4 = 3
4 ÷ 1 = 4	8 ÷ 2 = 4	12 ÷ 3 = 4	16 ÷ 4 = 4
5 ÷ 1 = 5	10 ÷ 2 = 5	15 ÷ 3 = 5	20 ÷ 4 = 5
6 ÷ 1 = 6	12 ÷ 2 = 6	18 ÷ 3 = 6	24 ÷ 4 = 6
7 ÷ 1 = 7	14 ÷ 2 = 7	21 ÷ 3 = 7	28 ÷ 4 = 7
8 ÷ 1 = 8	16 ÷ 2 = 8	24 ÷ 3 = 8	32 ÷ 4 = 8
9 ÷ 1 = 9	18 ÷ 2 = 9	27 ÷ 3 = 9	36 ÷ 4 = 9
10 ÷ 1 = 10	20 ÷ 2 = 10	30 ÷ 3 = 10	40 ÷ 4 = 10
11 ÷ 1 = 11	22 ÷ 2 = 11	33 ÷ 3 = 11	44 ÷ 4 = 11
12 ÷ 1 = 12	24 ÷ 2 = 12	36 ÷ 3 = 12	48 ÷ 4 = 12

5 ÷ 5 = 1	6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1
10 ÷ 5 = 2	12 ÷ 6 = 2	14 ÷ 7 = 2	16 ÷ 8 = 2
15 ÷ 5 = 3	18 ÷ 6 = 3	21 ÷ 7 = 3	24 ÷ 8 = 3
20 ÷ 5 = 4	24 ÷ 6 = 4	28 ÷ 7 = 4	32 ÷ 8 = 4
25 ÷ 5 = 5	30 ÷ 6 = 5	35 ÷ 7 = 5	40 ÷ 8 = 5
30 ÷ 5 = 6	36 ÷ 6 = 6	42 ÷ 7 = 6	48 ÷ 8 = 6
35 ÷ 5 = 7	42 ÷ 6 = 7	49 ÷ 7 = 7	56 ÷ 8 = 7
40 ÷ 5 = 8	48 ÷ 6 = 8	56 ÷ 7 = 8	64 ÷ 8 = 8
45 ÷ 5 = 9	54 ÷ 6 = 9	63 ÷ 7 = 9	72 ÷ 8 = 9
50 ÷ 5 = 10	60 ÷ 6 = 10	70 ÷ 7 = 10	80 ÷ 8 = 10
55 ÷ 5 = 11	66 ÷ 6 = 11	77 ÷ 7 = 11	88 ÷ 8 = 11
60 ÷ 5 = 12	72 ÷ 6 = 12	84 ÷ 7 = 12	96 ÷ 8 = 12
9 ÷ 9 = 1	10 ÷ 10 = 1	11 ÷ 11 = 1	12 ÷ 12 = 1
18 ÷ 9 = 2	20 ÷ 10 = 2	22 ÷ 11 = 2	24 ÷ 12 = 2
27 ÷ 9 = 3	30 ÷ 10 = 3	33 ÷ 11 = 3	36 ÷ 12 = 3
36 ÷ 9 = 4	40 ÷ 10 = 4	44 ÷ 11 = 4	48 ÷ 12 = 4
45 ÷ 9 = 5	50 ÷ 10 = 5	55 ÷ 11 = 5	60 ÷ 12 = 5
54 ÷ 9 = 6	60 ÷ 10 = 6	66 ÷ 11 = 6	72 ÷ 12 = 6
63 ÷ 9 = 7	70 ÷ 10 = 7	77 ÷ 11 = 7	84 ÷ 12 = 7
72 ÷ 9 = 8	80 ÷ 10 = 8	88 ÷ 11 = 8	96 ÷ 12 = 8
81 ÷ 9 = 9	90 ÷ 10 = 9	99 ÷ 11 = 9	108 ÷ 12 = 9
90 ÷ 9 = 10	100 ÷ 10 = 10	110 ÷ 11 = 10	120 ÷ 12 = 10
99 ÷ 9 = 11	110 ÷ 10 = 11	121 ÷ 11 = 11	132 ÷ 12 = 11
108 ÷ 9 = 12	120 ÷ 10 = 12	132 ÷ 11 = 12	144 ÷ 12 = 12

39. If 7 cents were divided as equally as possible among 3 boys, each boy would receive 2 cents, and there would be 1 cent left, or *remaining* undivided.

The number left after dividing, is called the **remainder**.

NOTES. — 1. Since the remainder is a part of the dividend, it must be of the same denomination. If the dividend is dollars, the remainder will be dollars; if pounds, the remainder will be pounds.

2. The remainder is always *less* than the divisor; for, if it were equal to it, or greater, the divisor would be contained at least once more in the dividend.

SHORT DIVISION

40. When the division is performed mentally, and merely the result is written, it is termed **short division**. Short division is used when the divisor does not exceed 12.

1. How many times is 2 contained in 468?

Here, the dividend is composed of three parts; 4 hundreds, 6 tens, and 8 units; that is, of 400, 60, and 8.

	Divisor		Dividend		Quotient	
	2	in	400	is contained	200	times.
	2	in	60	is contained	30	times.
	2	in	8	is contained	4	times.
Hence,	2	in	468	is contained	234	times.

The same result can be obtained without actually separating the dividend into parts:

Thus, 2 in 4 (hundreds), 2 times, which write in hundreds' place; 2 in 6 (tens), 3 times, which write in tens' place; 2 in 8 (units), 4 times, which write in units' place.

$$\begin{array}{r} \text{Dividend} \\ \text{Divisor, } 2\overline{)468} \\ \text{Quotient, } 234 \end{array}$$

2. How many times is 3 contained in 693? 231.
3. How many times is 4 contained in 848? 212.
4. How many times is 2 contained in 4682? 2341.
5. How many times is 4 contained in 8408? 2102.
6. How many times is 3 contained in 36936? 12312.
7. How many times is 2 contained in 88468? 44234.

41. 1. How many times is 3 contained in 129?

Solution. — Here, 3 is not contained in 1; but 3 is contained in 12 (tens), 4 (tens) times; write 4 in tens' place; 3 is contained in 9 (units), 3 times, which write in units' place.

$$\begin{array}{r} \text{OPERATION} \\ 3\overline{)129} \\ 43 \end{array}$$

2. How many times is 3 contained in 735?

SOLUTION.—Here, 3 is contained in 7 (hundreds), 2 (hundreds) times, and 1 hundred over; the 1 hundred, united with the 3 tens, makes 13 (tens), in which 3 is contained 4 (tens) times and 1 ten left; this 1 ten, united with the 5 units, makes 15 units, in which 3 is contained 5 times.

OPERATION

$$3\overline{)735}$$
$$245$$

3. How many times is 3 contained in 618?

SOLUTION.—Here, 3 is contained in 6 (hundreds), 2 (hundreds) times; as the 1 in tens' place will not contain 3, a cipher is placed in tens' place in the quotient; the 1 ten is then added to the 8 units, making 18 units, and the quotient figure 6 is placed in units' place.

OPERATION

$$3\overline{)618}$$
$$206$$

4. How many times is 3 contained in 609?

Here, the solution is the same as in the above example; there being no tens, their order is indicated by 0.

OPERATION

$$3\overline{)609}$$
$$203$$

5. How many times is 3 contained in 743?

After dividing, there is 2 left, the division of which is merely indicated by placing the divisor under the remainder; thus, $\frac{2}{3}$. The quotient is written thus, $247\frac{2}{3}$; read, 247, and *two divided by three;* or, 247, with a *remainder, two.*

OPERATION

$$3\overline{)743}$$
$$247\tfrac{2}{3}$$

6. How many times is 3 contained in 462? 154.

7. How many times is 5 contained in 1170? 234.

8. How many times is 4 contained in 948? 237.

Rule.—1. *Write the divisor at the left of the dividend, with a curved line between them, and draw a line beneath the dividend. Begin at the left hand, divide successively each figure of the dividend by the divisor, and write the result in the same order in the quotient.*

2. *If there is a remainder after dividing any figure, prefix it to the figure in the next lower order, and divide as before.*

3. *If the number in any order does not contain the divisor, place a cipher in the same order in the quotient, prefix the number to the figure in the next lower order, and divide as before.*

4. *If there is a remainder after dividing the last figure, place the divisor under it, and annex it to the quotient.*

Proof. — *Multiply the quotient by the divisor, and add the remainder, if any, to the product. If the work is correct, the sum will be equal to the dividend.*

Note. — This method of proof depends on the principle (§ 36) that a dividend is a product, of which the divisor and quotient are factors.

9. Divide 653 cents by 3.

SOLUTION		PROOF	
		217	
	Dividend	3	
Divisor,	3)653	651	= cents divided.
Quotient,	$217\frac{2}{3}$	2	= remainder.
		653	= dividend.

	10.	11.	12.
	6)454212	7)874293	8)3756031
	75702	124899	$469503\frac{7}{8}$
	6	7	8
Proof,	454212	874293	3756031

Parts of Numbers

When any number is divided into two equal parts, one of the parts is called *one half* of that number.

If divided into three equal parts, one of the parts is called *one third;* if into four equal parts, *one fourth;* if into five equal parts, *one fifth;* and so on.

Hence, to find *one half* of a number, divide by 2; to find *one third*, divide by 3; to find *one fourth*, divide by 4; to find *one fifth*, by 5, etc.

13. Divide 8652 by 2. 4326.

14. Divide 406235 by 3. $135411\frac{2}{3}$.

15. Divide 675043 by 4. $168760\frac{3}{4}$.

16. Divide 984275 by 5. 196855.

17. Divide 258703 by 6. $43117\frac{1}{6}$.

18. Divide 8643275 by 7. $1234753\frac{4}{7}$.

19. Divide 6032520 by 8. 754065.

20. Divide 9032706 by 9. 1003634.

21. Divide 1830024 by 10. $183002\frac{4}{10}$.

22. Divide 603251 by 11. 54841.

23. Divide 41674008 by 12. 3472834.

24. If oranges cost 3 cents each, how many can be bought for 894 cents? 298.

25. If 4 bushels of apples cost 140 cents, what is the price per bushel? 35 ¢.

26. If flour costs $4 a barrel, how many barrels can be bought for $812? 203.

27. A carpenter receives $423 for 9 months' work. How much is that a month? $47.

28. There are 12 months in 1 year. How many years are there in 540 months? 45.

29. There are 4 quarts in 1 gallon. How many gallons are there in 321276 quarts? 80319.

30. At $8 apiece, how many chairs can be bought for $1736? 217.

31. There are 7 days in one week. How many weeks are there in 734566 days? 104938.

32. A number has been multiplied by 11, and the product is 495. What is the number? 45.

33. The product of two numbers is 3582; one of the numbers is 9. What is the other? 398.

34. Find one half of 56. 28.

35. Find one half of 3725. $1862\frac{1}{2}$.

36. Find one third of 147. 49.

37. Find one fourth of 500. 125.

38. Find one fifth of 1945. 389.

39. Find one sixth of 4476. 746.

40. Find one seventh of 2513. 359.

41. Find one eighth of 5992. 749.

42. Find one ninth of 8793. 977.

43. Find one tenth of 1090. 109.

44. Find one eleventh of 4125. 375.

45. Find one twelfth of 5556. 463.

46. I divided 144 apples equally among 4 boys; the eldest boy gave one third of his share to his sister. What number did the sister receive? 12.

47. James found 195 cents, and gave to Daniel one fifth of them. Daniel gave one third of his share to his sister. How many cents did she receive? 13.

48. One eleventh of 275 is how much greater than one eighth of 192? 1.

LONG DIVISION

42. When the entire work of the division is written down, it is termed **long division.**

Long division is commonly used when the divisor exceeds 12.

1. Divide 3465 dollars equally among 15 men.

SOLUTION. — 15 is not contained any (thousands) times in 3 (thousands); therefore, there will be no thousands in the quotient. Take 34 (hundreds) as a *partial* dividend; 15 is contained in 34 (hundreds), 2 (hundreds) times; that is, 15 men have 2 hundred dollars each, which requires in all $15 \times 2 = 30$ hundreds of dollars.

OPERATION

```
15)3465(231
   30 hund.
   --
    46 tens.
    45
    --
     15 units.
     15
     --
```

Subtract 30 hundreds from 34 hundreds, and 4 hundreds remain; to which bring down the 6 tens, and you have 46 (tens) for a second partial dividend.

46 (tens) contains 15, 3 (tens) times; that is, each man has 3 ten dollars more, and all require $15 \times 3 = 45$ tens of dollars.

Subtract 45, and bring down the 5 units, which gives 15 (units) for a third partial dividend; in this the divisor is contained once, giving to each man 1 dollar more.

Hence, each man receives 2 hundred dollars, 3 ten dollars, and 1 dollar; that is, 231 dollars.

By this process, the dividend is separated into parts, each part containing the divisor a certain number of times.

Divisor	Parts	Quotients
15	3000	200
	450	30
	15	1
	3465	231

The first part, 30 hundreds, contains the divisor 2 (hundreds) times; the second part, 45 tens, contains it 3 (tens) times; the third part, 15 units, contains it 1 time.

The several parts together equal the given dividend, and the several partial quotients make up the entire quotient.

2. In 147095 days, how many years, of 365 days, are there?

SOLUTION. — Taking 147 (thousands) for the first partial dividend, we find it will not contain the divisor any thousands times; hence we find how many hundreds times 365 is contained in 1470 (hundreds).

OPERATION

```
365)147095(403 years.
    1460
    ----
      1095
      1095
      ----
```

Again, after multiplying and subtracting, as in the preceding example, and bringing down the 9 tens, the partial dividend, 109 (tens), will not contain the divisor any tens times; hence, write a cipher (no tens) in the quotient, and bring down the 5 units; the last partial dividend is 1095 (units), which contains the divisor three times.

3. Divide 4056 by 13. 312.

Rule. — 1. *Place the divisor on the left of the dividend, draw a curved line between them, and another on the right of the dividend.*

2. *Find how many times the divisor is contained in the fewest left-hand figures of the dividend that will contain the divisor, and place this number in the quotient at the right.*

3. *Multiply the divisor by this quotient figure; place the product under that part of the dividend from which it was obtained.*

4. *Subtract this product from the figures above it; to the remainder bring down the next figure of the dividend, and divide as before, until all the figures of the dividend are brought down.*

5. *If, at any time, after bringing down a figure, the number thus formed is too small to contain the divisor, place a cipher in the quotient, and bring down another figure, after which divide as before.*

Proof. — *Same as in short division.*

NOTES.—1. The product must never be *greater* than the partial dividend from which it is to be subtracted; when so, the quotient figure is *too large*, and must be diminished.

2. After subtracting, the remainder must always be *less* than the divisor; when the remainder is not less than the divisor, the last quotient figure is *too small*, and must be increased.

3. The order of each quotient figure is the same as the lowest order in the partial dividend from which it was obtained.

4. Divide 78994 by 319.

SOLUTION

```
319)78994(247 201/319
    638
    ----
    1519
    1276
    ----
     2434
     2233
     ----
      201, remainder.
```

PROOF

```
      247, quotient.
      319, divisor.
     ----
     2223
     247
    741
    -----
    78793
Add   201, remainder.
    -----
    78994 = the dividend.
```

5. Divide	11577 by 14.	$826\frac{13}{14}$.
6. Divide	48690 by 15.	3246.
7. Divide	1110960 by 23.	$48302\frac{14}{23}$.
8. Divide	122878 by 67.	1834.
9. Divide	12412 by 53.	$234\frac{10}{53}$.
10. Divide	146304 by 72.	2032.
11. Divide	47100 by 54.	$872\frac{12}{54}$.
12. Divide	71104 by 88.	808.
13. Divide	43956 by 66.	666.
14. Divide	121900 by 99.	$1231\frac{31}{99}$.
15. Divide	25312 by 112.	226.
16. Divide	381600 by 123.	$3102\frac{54}{123}$.
17. Divide	105672 by 204.	518.

18. Divide 600000 by 1234. $486\frac{276}{1234}$.

19. Divide 1234567 by 4321. $285\frac{3082}{4321}$.

20. Divide 50964242 by 7819. 6518.

21. Divide 48905952 by 9876. 4952.

22. Divide 4049160 by 12345. 328.

23. Divide 552160000 by 973. $567482\frac{14}{973}$.

24. At $15 an acre, how many acres of land can be bought for $3465? 231 acres.

25. If a man travels 26 miles a day, in how many days will he travel 364 miles? 14 days.

26. If $1083 is divided equally among 19 men, how many dollars will each have? $57.

27. A man raised 9523 bushels of corn on 107 acres. How much was that on one acre? 89 bu.

28. In one hogshead there are 63 gallons. How many hogsheads are there in 14868 gallons? 236.

29. The President receives $50000 a year (365 days). How much is that a day? $136 and $360 over.

30. The yearly income from a certain railroad is $379600. How much is that a day? (365 da.=1 yr.). $1040.

31. The product of two numbers is 6571435; one of the factors is 1235. What is the other? 5321.

32. Divide one million two hundred forty-seven thousand four hundred by four hundred five. 3080.

33. Divide ten million four hundred one thousand by one thousand six. $10338\frac{972}{1006}$.

34. A colony of 684 men bought a tract of land, containing 109440 acres. If the tract was equally divided among the purchasers, to how many acres was each man entitled? 160 acres.

35. A farmer raised 8288 bushels of corn, averaging 56 bushels to the acre. How many acres did he plant? 148 acres.

36. If a donation of $262275 is divided equally among 269 school libraries, how much will each receive? $975.

37. The earth, at the equator, is about 24899 miles in circumference, and turns on its axis once in 24 hours. How many miles an hour does it turn? $1037\frac{11}{24}$ mi.

38. A railroad 238 miles long, cost $3731840. What was the cost per mile? $15680.

39. A fort is 27048 feet distant from the city; the flash of a cannon was seen 24 seconds before the sound was heard. How many feet a second did the sound travel? 1127 feet.

40. Light travels at the rate of 11520000 miles a minute. How many minutes does it require for the light of the sun to reach the earth, the sun being 92160000 miles distant? 8 minutes.

41. Subtract 86247 from 94231 and divide the remainder by 16. 499.

42. Divide the sum of 46712 and 6848 by 104. 515.

43. Divide the product of 497 × 583 by 71. 4081.

44. To the difference between 2832 and 987 add 678, and divide the sum by 87. 29.

45. Multiply the difference between 4896 and 2384 by 49, and divide the product by 112. 1099.

46. Multiply the sum of 228 and 786 by 95, and divide the product by 114. 845.

47. Multiply the sum of 478 and 296 by their difference, and divide the product by 387. 364.

48. A horse dealer received $7560 for horses; he sold a part of them for $3885. If he sold the rest for $175 apiece, how many horses did he sell in the second lot?
21 horses.

49. A farmer expended at one time $7350 for land, and at another, $4655, paying $49 an acre each time. How many acres did he buy in both purchases? 245 acres.

50. A horse dealer bought 58 horses at $77 each, and sold them for $5742. How much did he gain on each horse? $22.

51. A man bought 240 acres of land, at $26 an acre, giving in payment a house valued at $2820, and horses valued at $180 apiece. How many horses did he give?
19 horses.

52. A speculator bought 25 acres of land for $10625, and after dividing it into 125 village lots, sold each lot for $250. How much did he gain on the whole? On each acre? On each lot? $20625. $825. $165.

CONTRACTIONS IN DIVISION

43. When the divisor can be separated into factors:

1. A man paid $255 for 15 acres of land. How much was that per acre?

SOLUTION. — 15 acres are 3 times 5 acres; dividing $255 by 3 gives $85, the value of 5 acres; dividing $85 by 5 gives $17, the value of 1 acre.

OPERATION

Dollars.
3 | 255 = the value of 15 acres.
5 | 85 = the value of 5 acres.
17 = the value of 1 acre.

The solution shows that instead of dividing by the number 15, whose factors are 3 and 5, we may first divide by one factor, then divide the quotient thus obtained by the other factor.

2. Find the quotient of 37, divided by 14.

SOLUTION.—Dividing by 2, the quotient is 18 and 1 unit remaining. That is, 37 = 18 *twos* and 1 unit remaining. Dividing by 7, the quotient is 2, with a remainder of 4 *twos*; the whole remainder then is 4 *twos* plus 1, or 9.

OPERATION

2 | 37
7 | 18 and 1 over.
  2 and 4 *twos* left.

Rule.—1. *Divide the dividend by one of the factors of the divisor; then divide the quotient thus obtained by the other factor.*

2. *Multiply the last remainder by the first divisor; to the product add the first remainder; the amount will be the true remainder.*

NOTE.—When the divisor can be resolved into more than two factors, you may divide by them successively. The true remainder will be found by multiplying each remainder by all the preceding divisors, except that which produced it. To their sum add the remainder from the first divisor.

Divide:

3.	2583 by 63.	$(63 = 7 \times 9)$	41
4.	6976 by 32.	$(32 = 4 \times 8)$	218.
5.	2744 by 28.	$(28 = 7 \times 4)$	98.
6.	6145 by 42.	$(42 = 6 \times 7)$	$146\frac{13}{42}$.
7.	19008 by 132.		144.
8.	7840 by 64.		$122\frac{32}{64}$.
9.	14771 by 72.		$205\frac{11}{72}$.
10.	10206 by 81.		126.
11.	81344 by 121.		$672\frac{32}{121}$.
12.	98272 by 108.		$909\frac{100}{108}$.

44. To divide by 1 with ciphers annexed; as, 10, 100, 1000, etc.:

To multiply 6 by 10, annex one cipher; thus, 60. On the principle that division is the reverse of multiplication, to divide 60 by 10, *cut off* a cipher.

NOTE. — Had the dividend been 65, the 5 might have been separated in the same manner as the cipher; 6 being the quotient, 5 the remainder. The same will apply when the divisor is 100, 1000, etc.

Rule. — *Cut off as many figures from the right of the dividend as there are ciphers in the divisor; the figures cut off will be the remainder, the other figures the quotient.*

1. Divide 34872 by 100.

OPERATION

1|00) 348|72
348, quo. 72, rem.

Divide:

2. 2682 by 10.	$268\frac{2}{10}$.	5. 46250 by 100.	$462\frac{50}{100}$.
3. 4700 by 100.	47.	6. 18003 by 1000.	$18\frac{3}{1000}$.
4. 37201 by 100.	$372\frac{1}{100}$.	7. 56055 by 1000.	$56\frac{55}{1000}$.

45. To divide when there are ciphers on the right of the divisor, or on the right of the divisor and dividend:

1. Divide 4072 by 800.

SOLUTION. — Regard 800 as composed of the factors 100 and 8, and divide as in the margin.

In dividing by 800, separate the two right-hand figures for the remainder, then divide by 8.

OPERATION

1|00 | 40|72
8 | 40
5, quo. 72, rem.

8|00) 40|72
5, quo. 72, rem.

2. Divide 77939 by 2400.

OPERATION

$$
\begin{array}{l}
24|00)779|39(32\frac{1139}{2400} \\
\underline{72} \\
\;59 \\
\;\underline{48} \\
\;11
\end{array}
$$

SOLUTION. — Since 2400 equals 24 × 100, cut off the two right-hand figures, the same as in dividing by 100; then divide by 24.

Dividing by 100, the remainder is 39; dividing by 24, the remainder is 11. To find the true remainder, multiply 11 by 100, and add 39 to the product (§ **43**, Rule); this is the same as annexing the figures cut off to the last remainder.

3. Divide 62700 by 2500.

OPERATION

$$
\begin{array}{l}
25|00)627|00(25\frac{200}{2500} \\
\underline{50} \\
127 \\
\underline{125} \\
\;\;2
\end{array}
$$

SOLUTION. — The same as for the example above.

Rule. — 1. *Cut off the ciphers at the right of the divisor, and as many figures from the right of the dividend.*

2. *Divide the remaining figures in the dividend by the remaining figures in the divisor.*

3. *Annex the figures cut off to the remainder, which gives the true remainder.*

Divide:

4.	73005 by 4000.	$18\frac{1005}{4000}$.
5.	36001 by 9000.	$4\frac{1}{9000}$.
6.	1078000 by 11000.	98.
7.	40167 by 180.	$223\frac{27}{180}$.
8.	907237 by 2100.	$432\frac{37}{2100}$.
9.	364006 by 6400.	$56\frac{5606}{6400}$.
10.	76546037 by 250000.	$306\frac{46037}{250000}$.
11.	43563754 by 63400.	$687\frac{7954}{63400}$.

GENERAL PRINCIPLES OF DIVISION

46. The value of the quotient depends on the relative values of divisor and dividend. These may be changed by multiplication and by division, thus:

1st. The dividend may be multiplied, or the divisor divided.

2d. The dividend may be divided, or the divisor multiplied.

3d. Both dividend and divisor may be multiplied, or both divided, at the same time.

Illustrations

Let 24 be a dividend, and 6 the divisor; the quotient is 4. $24 \div 6 = 4$.

If the dividend, 24, is *multiplied* by 2, the quotient will be multiplied by 2; for, $24 \times 2 = 48$; and $48 \div 6 = 8$, which is the former quotient, 4, *multiplied* by 2.

Now, if the divisor, 6, is *divided* by 2, the quotient will be multiplied by 2; for, $6 \div 2 = 3$; and $24 \div 3 = 8$, which is the former quotient, 4, *multiplied* by 2.

Principles.—I. *If the dividend is multiplied, or the divisor is divided, the quotient will be multiplied.*

Take the same example, $24 \div 6 = 4$.

If the dividend, 24, is *divided* by 2, the quotient will be divided by 2; for, $24 \div 2 = 12$; and $12 \div 6 = 2$, which is the former quotient, 4, *divided* by 2.

And, if the divisor, 6, is *multiplied* by 2, the quotient will be divided by 2; for, $6 \times 2 = 12$; and $24 \div 12 = 2$, which is the former quotient, 4, *divided* by 2.

II. *If the dividend is divided, or the divisor is multiplied, the quotient will be divided.*

Take the same example, $24 \div 6 = 4$.

If the dividend, 24, and divisor, 6, is *multiplied* by 2, the quotient will not be changed; for, $24 \times 2 = 48$; and $6 \times 2 = 12$; $48 \div 12 = 4$; the former quotient, 4, *unchanged.*

And if the dividend, 24, and divisor, 6, is *divided* by 2, the quotient will not be changed; for, $24 \div 2 = 12$; and $6 \div 2 = 3$; $12 \div 3 = 4$; the former quotient, 4, *unchanged.*

III. *If both dividend and divisor are multiplied or divided by the same number, the quotient will not be changed.*

Miscellaneous Examples

47. 1. In four bags there are \$500; in the first, \$96; in the second, \$120; in the third, \$55. What sum is in the fourth bag? \$229.

2. Four men paid \$1265 for land. The first paid \$243; the second \$61 more than the first; the third \$79 less than the second. How much did the fourth man pay? \$493.

3. I have five apple trees. The first bears 157 apples; the second, 264; the third, 305; the fourth, 97; the fifth, 123. I sell 428, and 186 are stolen. How many apples are left? 332.

4. In an army of 57068 men, 9503 are killed; 586 join the enemy; 4794 are prisoners; 1234 die of wounds; 850 are drowned. How many return? 40101.

5. On the first of the year a speculator is worth \$12307. During the year he gains \$8706; in January he spends \$237; in February, \$301; in each of the remaining ten months he spends \$538. How much has he at the end of the year? \$15095.

6. The Bible has 31173 verses. In how many days can I read it, by reading 86 verses a day? $362\frac{41}{86}$.

7. I bought 28 horses for $1400, but 3 died. For how much each must I sell the rest to incur no loss? $56.

8. How many times can I fill a 15-gallon cask from 5 hogsheads of 63 gallons each? 21 times.

9. A certain dividend is 73900; the quotient is 214; the remainder, 70. What is the divisor? 345.

10. Multiply the sum of 148 and 56 by their difference; divide the product by 23. 816.

11. How much cloth, at $6 a yard, will it take to pay for 8 horses at $60 each, and 14 cows at $45 each? 185 yd.

12. Two men paid $6000 for a farm; one man took 70 acres at $30 an acre, the other the remainder at $25 an acre. How many acres were there in all? 226.

13. My income is $1800 a year. If I spend $360 a year for provisions, $300 for rent, $150 for clothing, $100 for books, and $90 for incidentals, in how many years can I save $10400? 13.

14. A man bought 40 acres of ground at $15 an acre, and 80 acres at $25 an acre. He sold 90 acres for $4500, and the remainder at $60 an acre. For how much did he sell the whole land? How much did he gain?

$6300. $3700.

15. A merchant bought 275 yards of cloth at $4 a yard; he sold 250 yards at $5 a yard, and the remainder at $6 a yard. How much did he gain? $300.

16. A merchant having bought 125 sets of furniture at $85 a set, and 75 sets at $115 a set, decides to exchange them all for finer sets at $175 a set. How many of the latter does he get? 110.

17. A farmer sends to a dealer 20 horses and 15 mules to be sold. The dealer sells the horses for $125 each, and the mules for $150 each, charging $95 for selling. The farmer then buys 50 head of cattle at $45 each, with part of the money, and deposits the remainder in bank. How much does he deposit in bank? $2405.

AVERAGES

48. 1. A student passed his examination with 94% in arithmetic; 80% in history; 88% in geography; and 90% in grammar. What was his average per cent? 88%.

NOTE.—The **average** of two or more quantities is their sum divided by the number of quantities added. The sign % means per cent.

2. I mix 4 pounds of tea, worth 40¢ a pound, with 6 pounds, worth 50¢ a pound. What is 1 pound of the mixture worth?

SOLUTION.—4 pounds at 40¢ per pound are worth 160¢, and 6 pounds at 50¢ are worth 300¢; then, $4 + 6 = 10$ pounds are worth 460¢; hence, 1 pound costs $\frac{1}{10}$ of 460¢, or 46¢.

OPERATION

$$\begin{array}{l} 4 \times 40¢ = 160¢ \\ \underline{6 \times 50¢ = 300¢} \\ 10 \quad\quad)460¢ \\ \quad\quad\quad\quad 46¢ \end{array}$$

3. A drover bought 10 sheep, at $5 each, 15, at $6 each, and 15, at $4 each. What was the average price of the sheep? $5.

4. If a grocer mixes 8 pounds of tea, worth 60¢ a pound, with 2 pounds, at 110¢ a pound, and 6 pounds, at 70¢ a pound, how much is a pound of the mixture worth? 70¢.

5. Mix 6 pounds of sugar, at 3¢ a pound, with 4 pounds, at 8¢ a pound. How much will 1 pound of the mixture be worth? 5¢.

6. Mix 25 pounds sugar, at 12¢ a pound, 25 pounds, at 18¢, and 40 pounds, at 25¢. How much is 1 pound of the mixture worth? $19\frac{4}{9}$¢.

7. A mixes 3 gallons water, with 12 gallons vinegar, at 50¢ a gallon. How much is 1 gallon of the mixture worth? 40¢.

8. I have 30 sheep; 10 are worth \$3 each; 12, \$4 each; the rest, \$9 each. Find the average value. \$5.

9. On seven successive days the average temperature in Boston was 71°, 78°, 83°, 84°, 81°, 93°, 91°. What was the average temperature for the week? 83°.

10. The register of attendance in a grammar school for Monday was 995; for Tuesday, 1040; for Wednesday, 1080; for Thursday, 1020; for Friday, 1075. What was the average daily attendance for that week? 1042.

11. In a certain high school class 20 of the pupils are 14 years old, 20 are 15 years old, and 20 are 16 years old. What is the average age of the pupils? 15 years.

12. If a child spends 20 minutes on Monday preparing his arithmetic lesson, 30 minutes on Tuesday, 25 minutes on Wednesday, 45 minutes on Thursday, and 20 minutes on Friday, what is the average time per day spent during the week? 28 min.

13. What is the average scholarship in a class in which 5 pupils average 90% each, 10 average 95% each, and 10 average 80% each? 88%.

ORDER OF SIGNS

49. When several quantities are to undergo the same operation, we may inclose them in parentheses, ( ), in braces, { }, in brackets, [ ], or we may place them under a vinculum, $\overline{\quad}$. The quantities are then said to be *in parenthesis*. The signs are called **signs of aggregation.** They show that the quantities included by them are to be treated as a single number.

Thus, $(4 + 3) \times 8$ or $\overline{4 + 3} \times 8$ shows that the sum of 4 and 3 is to be multiplied by 8. 4 and 3 are in parenthesis.

The parts of an expression connected by the signs + or − are called **terms** of the expression.

Thus the expression $8 + 5 - 2$ contains three terms. $(5 - 2) \times 4 + (8 - 6) \times 2$ contains only two terms.

A quantity in parenthesis must be changed to a single quantity by performing the operations indicated. When there is one parenthesis within another, the inside one should be first removed, and then the next, until none remains. When a number is placed before a parenthesis, brace, or bracket without any sign, the sign × is understood.

Thus, $5(3 + 2)$ means $5 \times (3 + 2)$.

Precedence is given to the signs × and ÷ over the signs + and −; hence, where there is no parenthesis, the operations of multiplication and division should always be performed before addition and subtraction.

Thus, $4 + 8 \div 2 = 8$. $(4 + 8) \div 2 = 6$.

Find the value of:

1. $14[2+8\{6+(4+8)\div 2+(5\times 8)+\overline{14-2}\}+6-4]=$

(1) $14[2+8\{6+6+40+12\}+6-4]=$

(2) $14[2+512+6-4]=$

(3) $14\times 516=7224$, *Ans.*

EXPLANATION. — Remove the inner parentheses, ( ), and the vinculum, ——, by performing the operations indicated. $(4+8)\div 2=6$; $(5\times 8)=40$; $\overline{14-2}=12$. This gives (1). Remove the braces, { }. $\{6+6+40+12\}=64$; 64 multiplied by $8=512$. This gives (2). Remove the brackets, [ ]. $[2+512+6-4]=516$. This gives (3). 14 times $516=7224$, *Ans.*

Rule. — *Remove all the expressions in parenthesis by performing the operations indicated, beginning with the inner parenthesis. The answer to the last operation indicated will be the value of the expression.*

Find the value of:

2.	$(2+3)\times(7-4)$.	15.
3.	$(6+8)\div(10-3)$.	2.
4.	$[75-7\times 3+(4\times 4)-6]\div 8$.	8.
5.	$5\times[13+2(3+4\times 6)+5]$.	360.
6.	$\{200-\overline{8\times 8}+(3\times 9)-8\}\div 5$.	31.
7.	$8\times(96-26)\times\overline{5\times 6}-13\times 30(5\times 4)$.	9000.
8.	$[84-(7\times 6)+(3\times 5)-3]\div 9$.	6.
9.	$25\times(6\times 3)\times 4-(\overline{9\times 8}+90)$.	1638.
10.	$(54-16)\times\overline{11+4}-(15\times 20)$.	270.
11.	$10\times\{16-4+3(2+8-2)+3\times 6(4\div 2)+8\}\div 10$.	80.

COMPOUND DENOMINATE NUMBERS

50. A **denominate number** is a number composed of one or more units of a measure established by law or custom.

Thus, 8 dollars, 4 yards, 5 quarts, are denominate numbers.

A **simple denominate number** is one composed of units of one denomination only; a **compound denominate number** is one composed of units of two or more denominations of the same nature, that is, belonging to the same table.

Thus, 4 yards is a simple denominate number, but 4 yards, 2 feet, 4 inches is a compound denominate number.

NOTE. — Compound denominate numbers resemble simple numbers in the following particulars: the *denominations* correspond to the *orders* of simple numbers, and a certain number of units of a lower denomination make one unit of the next higher denomination. Most compound denominate numbers differ from simple numbers in this: ten units of each lower denomination do not uniformly make one unit of the next higher denomination. In United States Money and the Metric System of Weights and Measures, however, ten units of a lower denomination do make one unit of the next higher denomination.

51. **Reduction** is the process of changing the denomination of a number without altering its value.

Thus, 5 yards may be changed to feet; for, in 1 yard there are 3 feet; then, in 5 yards there are 5 times 3 feet, which are 15 feet.

Reduction takes place in two ways: 1st. From a higher denomination to a lower. 2d. From a lower denomination to a higher.

NOTE.— Reduction is little used in practical life. Articles are bought and sold in one denomination and its fractions. Thus, we buy goods in yards, half yards, quarter yards, eighth yards, not in yards, feet, and inches.

UNITED STATES MONEY

52. United States money is the money of the United States of America. The *unit* of United States money is the dollar.

TABLE

10 mills, (m).	= 1 cent,	marked ¢.
10 cents	= 1 dime,	marked d.
10 dimes	= 1 dollar,	marked \$.
10 dollars	= 1 eagle,	marked E.

The coins of the United States are:

Gold: Double eagle (\$20), eagle (\$10), half eagle (\$5), quarter eagle (\$2.50). (Gold dollars are no longer coined.)

Silver: Dollar, half dollar, quarter dollar, dime.

Nickel: Five-cent piece.

Bronze: One-cent piece.

NOTES.—1. The weight of the gold dollar is $25\frac{8}{10}$ grains Troy and of the silver dollar $412\frac{1}{2}$ grains Troy. The silver coin is nearly sixteen times the weight of the gold coin.

2. United States money was established, by Act of Congress, in 1786. The first money coined, by the authority of the United States, was in 1793. The character, \$, is supposed to be a contraction of U. S. (United States) the U being placed upon the S.

3. The mill is not coined. It is used only in calculations.

A sum of money is expressed as *dollars and cents*, and, when written in figures, is always preceded by the *dollar sign* (\$).

NOTE.—Calculations are sometimes carried out to mills, but, in business transactions, the final result is always taken to the nearest cent; that is, less than 5 mills is disregarded, and 5 mills or more are reckoned as 1 cent.

A period (.), called the **decimal point,** is used to separate the dollars and cents.

Eagles are read as tens of dollars, and dimes are read as tens of cents.

Thus, $24.56 is read 24 dollars 56 cents; *not* 2 eagles 4 dollars 5 dimes 6 cents. $16.375 is read 16 dollars 37 cents 5 mills.

The figures to the left of the decimal point express a number of dollars; the two figures to the right of the decimal point, a number of cents; and the third figure to the right, mills.

NOTE.—If the number of cents is less than 10, a cipher must be put in the tens' place, and the decimal point must be placed at the left of the cipher; thus, 5 cents is written $.05 or $0.05.

Write:

1. Twelve dollars seventeen cents eight mills.
2. Six dollars six cents six mills.
3. Seven dollars seven mills.
4. Forty dollars fifty-three cents five mills.
5. Two dollars three cents.
6. Twenty dollars two cents two mills.
7. One hundred dollars ten cents.
8. Two hundred dollars two cents.
9. Four hundred dollars one cent eight mills.

Read:

10.	$18.625	**11.**	$ 70.015	**12.**	$6.12	**13.**	$ 29.00
	$20.324		$100.28		$3.06		$100.03
	$79.05		$150.05		$4.31		$ 20.05
	$46.00		$100.00		$5.43		$ 40.125

Reduction of U.S. Money

53. As there are ten mills in 1 cent, in any number of cents there are 10 times as many mills as cents.

To reduce cents to mills:

Rule. — *Multiply the number of cents by ten; that is, annex one cipher.*

To reduce mills to cents:

Rule. — *Divide the number of mills by ten; that is, cut off one figure from the right.*

As there are 10 cents in one dime and 10 dimes in 1 dollar, there are $10 \times 10 = 100$ cents in 1 dollar; then, in any number of dollars there are 100 times as many cents as dollars.

To reduce dollars to cents:

Rule. — *Multiply the number of dollars by one hundred; that is, annex two ciphers.*

To reduce cents to dollars:

Rule. — *Divide the number of cents by one hundred; that is, cut off two figures from the right.*

As there are 10 mills in 1 cent and 100 cents in 1 dollar, there are $100 \times 10 = 1000$ mills in 1 dollar; then, in any number of dollars there are 1000 times as many mills as dollars.

To reduce dollars to mills:

Rule. — *Multiply the number of dollars by one thousand; that is, annex three ciphers.*

To reduce mills to dollars:

Rule. — *Divide the number of mills by one thousand; that is, cut off three figures from the right.*

54. The reduction of mills or cents to dollars may be made simply with the decimal point. Thus,

1st. **If the sum is mills:** **Rule.** — *Put the decimal point between the third and fourth figures from the right.*

2d. **If the sum is cents:** **Rule.** — *Put the decimal point between the second and third figures from the right.*

Reduce:

1.	17 ¢ to mills.	170 m.
2.	28 ¢ to mills.	280 m.
3.	43 ¢ and 6 m. to mills.	436 m.
4.	70 ¢ and 6 m. to mills.	706 m.
5.	106 mills to cents.	10 ¢ 6 m.
6.	490 mills to cents.	49 ¢.
7.	9 dollars to cents.	900 ¢.
8.	14 dollars to cents.	1400 ¢.
9.	104 dollars to cents.	10400 ¢.
10.	$60 and 13 ¢ to cents.	6013 ¢.
11.	$40 and 5 ¢ to cents.	4005 ¢.
12.	375 ¢ to dollars.	$3.75.
13.	9004 ¢ to dollars.	$90.04.
14.	4 dollars to mills.	4000 m.
15.	$14 and 2 ¢ to mills.	14020 m
16.	2465 mills to dollars.	$2.465.
17.	3007 mills to dollars.	$3.007.
18.	3187 ¢ to dollars.	$31.87.
19.	10375 mills to dollars.	$10.375.

Addition of U.S. Money

55. 1. Add 4 dollars 12 cents 5 mills, 7 dollars 6 cents 2 mills, 20 dollars 43 cents, 10 dollars 5 mills, 16 dollars 87 cents 5 mills.

Rules. — 1. *Write the numbers and add as in simple numbers.*

2. *Place the decimal point in the sum under the decimal points above.*

Proof. — *The same as in addition of simple numbers.*

OPERATION

$ ¢ m
4.125
7.062
20.430
10.005
16.875
$58.497

2. What is the sum of 17 dollars 15 cents, 23 dollars 43 cents, 7 dollars 19 cents, 8 dollars 37 cents, and 12 dollars 31 cents? $68.45.

3. Add 18 dollars 4 cents 1 mill, 16 dollars 31 cents 7 mills, 100 dollars 50 cents 3 mills, and 87 dollars 33 cents 8 mills. $222.199.

4. William had the following bills for collection: $43.75, $29.18, $17.63, $268.95, and $718.07. How much was to be collected? $1077.58.

5. A man bought a wagon for $200, a watch for $43.87, a suit of clothes for $56.93, a hat for $8.50, and a whip for $2.31. What was the amount of the bill? $311.61.

6. A man has due him $504.06, $420.19, $105.50, $304, $888.47. What is the whole amount due him? $2222.22.

7. Add $5.07, $30.203, $100.005, $60.02, $700.011, $1000.10, $40.004, and $64.587. $2000.

Subtraction of U.S. Money

56. 1. From one hundred dollars five cents three mills take eighty dollars twenty cents seven mills.

Rule. — 1. *Write the numbers and subtract as in simple numbers.*

2. *Place the decimal point in the remainder under the decimal points above.*

Proof. — *The same as in subtraction of simple numbers.*

OPERATION

$ ¢ m.
100.053
80.207
$19.846

2. From $29.342 take $17.265. $12.077.
3. From $46.28 take $17.75. $28.53.
4. From $20.05 take $5.50. $14.55.
5. From $3 take 3¢. $2.97.
6. From $10 take 1 mill. $9.999.
7. From $50 take 50¢ 5 mills. $49.495.
8. From one thousand dollars take one dollar one cent and one mill. $998.989.
9. B owes 1000 dollars 43 cents. If he pays $900.68, how much will he still owe? $99.75.

Multiplication of U.S. Money

57. 1. How much will 13 cows cost at 47 dollars 12 cents 5 mills each?

Rule. — 1. *Multiply as in simple numbers.*

2. *Put the decimal point in the same place in the product as it is in the multiplicand.*

Proof. — *The same as in multiplication of simple numbers.*

OPERATION

$47.125
13
141375
47125
$612.625

2. Multiply \$7.835 by 8. \$62.68.

3. Multiply \$12 9¢ 3 m. by 9. \$108.837.

4. Multiply \$23 1¢ 8 m. by 16. \$368.288.

5. Multiply \$35 14¢ by 53. \$1862.42.

6. Multiply \$125 2¢ by 62. \$7751.24.

7. Multiply \$40 4¢ by 102. \$4084.08.

8. Multiply 12¢ 5 m. by 17. \$2.125.

9. Multiply \$3.28 by 38. \$124.64.

10. Find the cost of 338 geographies at 1 dollar 6 cents each. \$358.28.

11. A man sold 38 chairs at 5 dollars 75 cents apiece. To how much did they amount? \$218.50.

12. At 7¢ a pound, how much will 465 pounds of rice cost?

NOTE. — Instead of multiplying 7 cents by 465, multiply 465 by 7, which gives the same product (§ 30). But to place the decimal point remember that 7 cents is the true multiplicand.

OPERATION

$$\begin{array}{r} 465 \\ .07 \\ \hline \$32.55 \end{array}$$

13. Find the cost of 178 yards of sheeting at 17¢ a yard. \$30.26.

14. Find the cost of 24 yards of velvet at \$5.67 a yard. \$136.08.

15. A merchant has 169 shirtwaists, valued at \$2.69 each. What is the value of all? \$454.61.

16. If I sell 691 chairs at \$1.25 each, how much will they amount to? \$863.75.

17. I sold 73 hogsheads of molasses, of 63 gallons each, at 55¢ a gallon. What was the amount of the bill? \$2529.45.

18. What will be the cost of 4 barrels of sugar, of 281 pounds each, at 6 cents 5 mills a pound? $73.06.

19. I bought 35 bolts of tape, of 10 yards each, at 1 cent a yard. How much did it cost? $3.50.

20. If I earn 13¢ an hour, and work 11 hours a day, how much shall I earn in 312 days? $446.16.

21. I sold 18 bags of wheat, of 3 bushels each, at $.92 a bushel. What was the amount of the bill? $49.68.

22. Find the cost of 150 acres of land, at 10 dollars 50¢ per acre. $1575.

23. Find the cost of 17 bags of coffee, of 51 pounds each, at 25¢ per pound. $216.75.

Division of U.S. Money

58. I. To find how many times one sum of money is contained in another:

1. How much cloth, at 7 cents a yard, will $1.75 buy?

OPERATION

$$\begin{array}{r} 7\underline{)175} \\ 25 \end{array}$$

SOLUTION.— As many yards as 7 cents is contained times in 175 cents, which are 25.

Rule. — 1. *Reduce both sums of money to the same denomination.*

2. *Divide as in simple numbers.*

2. How much rice, at 9 cents a pound, can be bought for 72 cents? 8 lb.

3. How many towels, at 37 cents and 5 mills apiece, can be bought for $6? 16.

4. How many yards of calico, at 8 cents a yard, can be bought for $2.80? 35 yd.

5. How many yards of ribbon, at 25 cents a yard, can be purchased for $3? 12 yd.

6. At $4.05 a barrel, how many barrels of flour will $81 purchase? 20 bbl.

7. At 5 cents each, how many oranges can be bought for $1.20? 24.

8. At $.905 per bushel, how many bushels of wheat can be purchased for $188.24? 208 bu.

II. To divide a sum of money into a given number of equal parts:

1. A man worked 3 days for $3.75. What were his daily wages?

OPERATION

SOLUTION. — His daily wages were $3.75 ÷ 3 = $1.25.

$$\begin{array}{r} 3)\underline{\$3.75} \\ \$1.25 \end{array}$$

2. A bookseller sold 6 histories for $9. How much did he get for each?

OPERATION

SOLUTION. — He got for each, $9 ÷ 6. $9 divided by 6 gives a quotient $1, with a remainder $3 = 300 cents. 300 cents divided by 6 gives a quotient 50 cents.

$$\begin{array}{r} 6)\underline{\$9.00} \\ \$1.50 \end{array}$$

Rule. — 1. *Divide as in simple numbers.*

2. *Put the decimal point in the same place in the quotient as it is in the dividend.*

NOTES. — 1. If the dividend is dollars, and the division not exact, annex two ciphers after the decimal point for cents; and, if necessary, a third cipher for mills.

2. Should there be a remainder after obtaining the mills, it may be indicated by the sign + placed after the quotient.

3. Divide $64.96 among 8 persons. $8.12.

4. A florist received $29.61 for 23 rosebushes. How much was that apiece? $1.287 +.

5. If 4 acres of land cost \$92.25, how much is that an acre? \$23.062+.

6. Divide \$57.44 among 8 persons. \$7.18.

7. A boy received \$25.76 for 16 days' work. How much was that for a day? \$1.61.

8. I bought 755 pounds of tea for \$328.425. How much did it cost a pound? \$0.435.

9. My salary is \$800 a year. How much is that a day, there being 313 working days in the year? \$2.555+.

10. Divide \$9310 among 133 men. What is each man's share? \$70.

11. A man purchased a farm of 154 acres for \$2704.24. How much did it cost per acre? \$17.56.

12. I sold 15 kegs of butter, of 25 pounds each, for \$75. How much was that a pound? 20 ¢.

13. I bought 8 barrels of sugar, of 235 pounds each, for \$122.20. How much did 1 pound cost? \$0.065.

Miscellaneous Examples

59. 1. I owe A \$47.50; B, \$38.45; C, \$15.47; D, \$19.43. What sum do I owe? \$120.85.

2. A owes \$35.25; B, \$23.75; C, as much as A and B, and \$1 more. How much do they all owe? \$119.

3. A paid me \$18.38; B, \$81.62; C, twice as much as A and B. How much did I receive? \$300.

4. I went to market with \$5. I spent for butter 75 cents, for eggs 35 cents, for vegetables 50 cents, for flour \$1.50. How much money was left? \$1.90.

5. I get \$50 a month, and spend \$30.50 of it. How much shall I have left in 6 months? \$117.

6. A lady had $20; she bought a dress for $8.10, shoes for $5.65, eight yards of linen at 25 cents a yard, and a shawl for $4. What sum was left? 25¢.

7. A farmer sold his corn for $21.75. He paid for sugar $3.85, for tea $1.25, for coffee $2.50, for spices $1.50. How much had he left? $12.65.

8. I owe A $37.06; B, $200.85; C, $400; D, $236.75, and E, $124.34; my property is worth $889.25. How much do I owe more than I am worth? $109.75.

9. I bought 143 yards of velveteen, at 23 cents a yard. After paying $12.60, how much did I owe? $20.29.

10. A owed me $400: he paid me 435 bushels of corn, at 45 cents a bushel. What sum is due me? $204.25.

11. If B spends 65 cents a day, how much will he save in 365 days, his income being $400? $162.75.

12. A man bought 63 bushels of oats at 35 cents a bushel. How much did they cost? $22.05.

13. Find the cost of 76 yards of lawn, at 23 cents a yard. $17.48.

14. If 25 men perform a piece of work for $2000, and spend, while doing it, $163.75, what will be each man's share of the profits? $73.45.

15. If 16 men receive $516 for 43 days' work, how much does each man earn a day? 75¢.

16. C earned $90 in 40 days, working 10 hours a day. How much did he earn an hour? 22¢ 5 m.

17. A merchant failing, has goods worth $1000, and $500 in cash, to be equally divided among 22 creditors. How much will each receive? $68.18+.

BILLS AND ACCOUNTS

60. A **bill** is a written dated statement of articles sold, or services rendered, with the price of each item and the entire cost.

The following are specimens of bills:

CINCINNATI, Ohio, June 1, 1903.

MR. FRANK HOLT,

1\. *Bought of* PHILIP ARDSLY.

May	10	10 lb. Coffee, @ 25¢	$2	50
	11	2 lb. Tea, @ 65¢	1	30
	19	10 lb. Sugar, @ 6¢		60
	27	10 lb. Rice, @ 8¢		80
		Received payment,	$5	20

PHILIP ARDSLY.

ARNOLD, GREEN & Co.
14th St. and 6th Ave.

NEW YORK, April 1, 1902.
MRS. B. ATWOOD, Dr.
34 W. 95th St.

2\.

March	5	1 Kettle	$3	00		
	10	1 Hat	8	00		
	17	5 quires Paper . . 18¢		90		
	19	2 doz. Buttons . . 5¢		10		
	21	1 pair Gloves . . .	1	65		
	28	6 yards Lawn . . 16¢		96		
	31	1 Waist . . .	15	75		
					$30	36
		Credits				
	19	5 quires Paper . . 18¢		90		
	22	1 pair Gloves . .	1	65	2	55
		Received payment,			$27	81

ARNOLD, GREEN & Co.
April 3, 1902, *per* McB.

Make out the following bills in proper form, supplying dates, name of seller and purchaser. Find the amounts.

3. Bought	9 pounds Coffee,	at	$0.25 per lb.	$
	4 pounds Tea,	at	.60 per lb.	
	45 pounds Sugar,	at	.06 per lb.	
	17 pounds Cheese,	at	.16 per lb.	
				$10.07

4. Bought	22 yards Silk,	at	$1.75 per yd.	$
	18 yards Muslin,	at	.15 per yd.	
	50 yards Linen,	at	.65 per yd.	
	6 yards Gingham,	at	.18 per yd.	
Credited	25 yards Linen,	at	.65 per yd.	
				$58.53

5. Bought	4 pounds Prunes,	at	$0.18 per lb.	$
	8 pounds Butter,	at	.23 per lb.	
	7 pounds Dates,	at	.11 per lb.	
	6 pounds Rice,	at	.09 per lb.	
	13 pounds Coffee	at	.35 per lb.	
	26 pounds Cheese	at	.12 per lb.	
				$11.54

61. Bookkeeping is a systematic method of recording business transactions.

A **debtor (Dr.)** is one who owes another money, goods, services, etc. A **creditor (Cr.)** is one to whom money, goods, or services are due.

An **account** is a statement of the debits and credits of a business transaction. The left-hand or debit side is marked **Dr.** and the right-hand or credit side **Cr.** The difference between the footings (sums) of the two sides is the **balance of the accounts.**

The following shows the form of an account:

FRANK BREWSTER *in account with* E. L. CLARK & CO.

510 Broadway, NEW YORK, N.Y.,

Jan. 1, 1903.

1. *Dr.* *Cr.*

1902					1902				
July	9	200 lb. Butter, @ $ .25	$50	00	Nov.	4	10 bbl. Apples, @ $3.00	$30	00
Aug.	15	305 lb. Cheese, @ .15	45	75	Nov.	29	Cash on Account,	50	00
Aug.	19	200 lb. Lard, @ .12½	25	00	Dec.	1	10 bbl. Apples, @ 2.75	27	50
Sept.	3	300 lb. Tallow, @ .15	45	00	Dec.	23	100 bu. Potatoes, @ .45	45	00
Sept.	18	100 doz. Eggs, @ .15	15	00	1903				
Dec.	15	210 lb. Butter, @ .25	52	50	Jan.	1	**Balance,**	**80**	**75**
			$233	25				$233	25

EXPLANATION. — The *Dr.* side shows that Mr. Brewster owes E. L. Clark & Co. $233.25 for groceries purchased. The *Cr.* side shows the goods delivered or money paid by Mr. Brewster to E. L. Clark & Co. Since the footing of the debit side exceeds that of the credit side by $80.75, a balance of $80.75 (marked in black type on the credit side) is due E. L. Clark & Co., from Mr. Brewster.

NOTE. — A balance on the credit side, or **credit balance**, shows how much money must be paid or credit given to balance the account, that is, how much is still due by the individual whose account is kept. A balance on the debit side, or **debit balance**, shows how much more the individual must owe to balance his credits, that is, what sum is to his credit.

Arrange in proper form and balance the following accounts:

2. March 1, 1902, Walter Briggs owes James Turner & Co., on account, a balance of $145.75. March 10, Briggs buys of Turner, on credit, 50 pieces of velveteen of 36 yd. each at $.35 a yard. March 17, he buys 25 lumps of velveteen of 150 yd. each at $.30 a yard. April 14, Briggs sends a check for $750 on account. April 17, Briggs buys of Turner & Co. 1000 yd. of cloth at $2 a yard. April 24, Briggs embosses for Turner & Co., on credit, 50 pieces of velveteen of 36 yd. each at $.01½ a yard.

Briggs owes Turner $3123.75.

3. April 1, 1902, Frank Owen owes Silas Graham a balance of $500. April 3, Owen sells Graham, on credit, merchandise amounting to $275. April 10, Owen pays Graham $200 cash on account. May 5, Owen sells Graham merchandise amounting to $175.65. May 15, Owen buys of Graham, on credit, 50 bu. potatoes at $.50 a bushel.

Graham owes Owen $125.65.

REDUCTION

DRY MEASURE

62. Dry measure is used in measuring grain, vegetables, fruit, coal, etc.

TABLE

2 pints (pt.)	= 1 quart, marked qt.
8 quarts	= 1 peck, marked pk.
4 pecks	= 1 bushel, marked bu.

NOTES.—1. The *standard unit* of dry measure is the bushel; it is a cylindrical measure $18\frac{1}{2}$ inches in diameter, 8 inches deep, and contains $2150\frac{2}{5}$ cubic inches.

2. All the denominations are used in trade, the peck being the least in use.

3. When articles usually measured by the above table are sold by weight, the *bushel* is taken as the unit. The number of avoirdupois pounds in a bushel varies in different states and with different articles. In most states there are 60 pounds in a bushel of beans, peas, clover seed, potatoes, and wheat; there are 56 pounds in a bushel of shelled corn, rye, and flaxseed, and 32 pounds in a bushel of oats.

1. How many pints are there in 2 quarts? In 4? In 6? In 8? In 10?

2. How many quarts are there in 3 pk.? In 5? In 7?

3. How many pecks are there in 9 bu.? In 11? In 13? In 15? In 17? In 19?

4. How many quarts are there in 10 bu.? In 12? In 14? In 18? In 25? In 56?

5. How many pecks are there in 16 qt.? In 24? In 32? In 40? In 48? In 64?

6. How many bushels are there in 32 qt.? In 64?

7. How many pints are there in 1 bu.? In 2? In 5?

The preceding examples show that to reduce quarts to pints we multiply the number of quarts by the number of pints in a quart; to reduce pecks to quarts we multiply the number of pecks by the number of quarts in a peck; and to reduce bushels to pecks we multiply the number of bushels by the number of pecks in a bushel.

I. To reduce from a higher to a lower denomination:

Rule.—*Multiply by the number of units of the lower denomination that make one unit of the higher denomination.*

They also show that to reduce pints to quarts we divide the number of pints by the number of pints in a quart; to

reduce quarts to pecks we divide the number of quarts by the number of quarts in a peck; and to reduce pecks to bushels we divide the number of pecks by the number of pecks in a bushel.

II. To reduce from a lower to a higher denomination:

Rule. — *Divide by the number of units of the lower denomination that make one unit of the higher denomination.*

1. Reduce 3 bushels to pints.

SOLUTION. — Since there are 4 pk. in 1 bu., in 3 bu. there are 3 times 4 pk., or 12 pk.

Since there are 8 qt. in 1 pk., in 12 pk. there are 12 times 8 qt., or 96 qt.

Since there are 2 pt. in 1 qt., in 96 qt. there are 96 times 2 pt., or 192 pt.

1ST OPERATION

3×4 pk. = 12 pk.
12×8 qt. = 96 qt.
96×2 pt. = 192 pt.

2D OPERATION

3, no. of bu.
4
12, no. of pk.
8
96, no. of qt.
2
192, no. of pt.

NOTE. — The explanation shows that 4 pk. is the first multiplicand and 3 the multiplier; 8 qt. the second multiplicand and 2 the multiplier; 2 pt. the third multiplicand and 96 the multiplier — as indicated in the 1st operation; but it is evident that much labor is saved by the 2d operation, and since (§ **30**) either factor may be considered as the multiplier when both are abstract, it is better in practice to regard the numbers as abstract and proceed as indicated in the 2d operation. This must not be interpreted as indicating that 3 bu. multiplied by 4 = 12 pk., which would, of course, be absurd.

2. Reduce 192 pints to bushels.

SOLUTION. — Since there are 2 pt. in 1 qt., in 192 pt. there are as many quarts as 2 pt. are contained times in 192 pt., which is 96 times; therefore the number of quarts is 96.

Since there are 8 qt. in 1 pk., in 96 qt. there are as many pecks as 8 qt. is contained times in 96 qt., which is 12 times; therefore the number of pecks is 12.

OPERATION

2 | 192, no. of pt.
8 | 96, no. of qt.
4 | 12, no. of pk.
3, no. of bu.

Since there are 4 pk. in 1 bu., in 12 pk. there are as many bushels as 4 pk. is contained times in 12 pk., which is 3 times; therefore the number of bushels is 3.

NOTE. — Regard dividend and divisor as abstract numbers. Do not interpret the operation to indicate 192 pt. ÷ 2 = 96 qt.

The two preceding examples show that *reduction from a higher to a lower denomination, and from a lower to a higher denomination, prove each other.*

3. Reduce 7 bu. 3 pk. 6 qt. 1 pt. to pints.

SOLUTION. — Since there are 4 pk. in 1 bu., in 7 bu. there are 7 times 4 pk., or 28 pk.; 28 pk. + 3 pk = 31 pk.

Since there are 8 qt. in 1 pk., in 31 pk. there are 31 times 8 qt., or 248 qt.; 248 qt. + 6 qt. = 254 qt.

Since there are 2 pt. in 1 qt., in 254 qt. there are 254 times 2 pt., or 508 pt.; 508 pt. + 1 pt. = 509 pt., the result.

OPERATION

bu.	pk.	qt.	pt.
7	3	6	1
4			
31, no. of pk. in 7 bu. 3 pk.			
8			
254, no. of qt. in 31 pk. 6 qt.			
2			
509, no. of pt. in the whole.			

4. Reduce 509 pt. to bushels.

SOLUTION. — Since there are 2 pt. in 1 qt., in 509 pt. there are as many quarts as 2 pt. are contained times in 509 pt., or 254 qt. 1 pt.

Since there are 8 qt. in 1 pk., in 254 qt. there are as many pecks as 8 qt. are contained times in 254 qt., or 31 pk. 6 qt.

OPERATION

2	509, no. of pt.
8	254, no. of qt. + 1 pt.
4	31, no. of pk. + 6 qt.
	7, no. of bu. + 3 pk.

Since there are 4 pk. in 1 bu., in 31 pk. there are as many bushels as 4 pk. are contained times in 31 pk. or 7 bu. 3 pk. The answer is, therefore, 7 bu. 3 pk. 6 qt. 1 pt.

63. I. To reduce from a higher to a lower denomination:

Rule. — 1. *Multiply the number of the highest denomination given, by that number of the next lower which makes a unit of the higher.*

2. *Add to the product the number, if any, of the lower denomination.*

3. *Proceed in like manner with the result thus obtained, till the whole is reduced to the required denomination.*

II. To reduce from a lower to a higher denomination:

Rule. — 1. *Divide the given quantity by the number of units of its own denomination which make a unit of the next higher.*

2. *Proceed in like manner with the quotient thus obtained, till the whole is reduced to the required denomination.*

3. *The last quotient, with the several remainders, if any, annexed, will be the answer.*

Proof. — *Reverse the operation, that is, reduce the answer to the denomination from which it was derived. If this result is the same as the quantity given, the work is correct.*

5. Reduce 4 bu. 2 pk. 1 qt. to pints. 290 pt.
6. Reduce 7 bu. 3 pk. 7 qt. 1 pt. to pints. 511 pt.
7. Reduce 3 bu. 1 pt. to pints. 193 pt.
8. Reduce 384 pt. to bushels. 6 bu.
9. Reduce 47 pt. to pecks. 2 pk. 7 qt. 1 pt.
10. Reduce 95 pt. to bushels. 1 bu. 1 pk. 7 qt. 1 pt.
11. Reduce 508 pt. to bushels. 7 bu. 3 pk. 6 qt.

LIQUID MEASURE

64. Liquid measure is used for measuring all liquids.

TABLE

4 gills (gi.)	=	1 pint, marked pt.
2 pints	=	1 quart, marked qt.
4 quarts	=	1 gallon, marked gal.

NOTE.—The *standard unit* of liquid measure is the *gallon*, which contains 231 cubic inches. All the denominations are in general use.

1. Reduce 17 gal. to pints. 136 pt.
2. Reduce 13 gal. to gills. 416 gi.
3. Reduce 126 gal. to pints. 1008 pt.
4. Reduce 1260 gal. to gills. 40320 gi.
5. Reduce 1120 gi. to gallons. 35 gal.
6. How many gallons are there in 1848 cubic inches? 8 gal.
7. How many gallons are there in a vessel containing 138138 cubic inches? 598 gal.

AVOIRDUPOIS WEIGHT

65. Avoirdupois weight is used for weighing all coarse and heavy articles like hay, grain, groceries, coal, etc.

TABLE

16 ounces (oz.)	= 1 pound, marked lb.
100 pounds	= 1 hundredweight, marked cwt.
20 cwt., or 2000 lb.,	= 1 ton, marked T.

NOTES.—1. The standard avoirdupois pound of the United States is determined from the Troy pound, and contains 7000 grains Troy.

2. At the Custom House (and in some trades) 2240 pounds are considered a ton, being called a long ton.

3. The ounce and the hundredweight are little used.

1. Reduce 2 cwt. to pounds. 200 lb.
2. Reduce 3 cwt. 75 lb. to pounds. 375 lb.
3. Reduce 1 T. 3 cwt. to pounds. 2300 lb.
4. Reduce 3 T. 75 lb. to pounds. 6075 lb.
5. Reduce 4 cwt. 44 lb. to pounds. 444 lb.

6. Reduce 5 T. 90 lb. to pounds. 10090 lb.

7. Reduce 2 cwt. 77 lb. 12 oz. to ounces. 4444 oz.

8. Reduce 2 cwt. 17 lb. 3 oz. to ounces. 3475 oz.

9. Reduce 1 T. 6 cwt. 4 lb. 2 oz. to ounces. 41666 oz.

10. Reduce 4803 lb. to cwt. 48 cwt. 3 lb.

11. Reduce 22400 lb. to tons. 11 T. 4 cwt.

12. Reduce 2048000 oz. to tons. 64 T.

13. Reduce 64546 oz. to cwt. 40 cwt. 34 lb. 2 oz.

14. Reduce 97203 oz. to tons. 3 T. 75 lb. 3 oz.

15. Reduce 544272 oz. to tons. 17 T. 17 lb.

16. What is the total weight of 52 parcels, each containing 18 lb.? 9 cwt. 36 lb.

17. What is the weight of 180 iron castings, each weighing 75 lb.? 6 T. 15 cwt.

LONG MEASURE

66. Long measure is used in measuring distances or length, in any direction.

TABLE

12	inches (in.)	= 1 foot, marked ft.
3	feet	= 1 yard, marked yd.
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ feet,	= 1 rod, marked rd.
320	rods	= 1 mile, marked mi.

NOTES.—1. The standard unit of length is the *yard*. The standard yard for the United States is preserved at Washington. A copy of this standard is kept at each state capital.

2. In trade the yard and foot are the denominations most in use.

1. Reduce 2 yd. 2 ft. 7 in. to inches. 103 in.

2. Reduce 7 yd. 11 in. to inches. 263 in.

3. Reduce 12 mi. to rods. 3840 rd.

4.	Reduce 7 mi. 240 rd. to rods.	2480 rd.
5.	Reduce 9 mi. 31 rd. to rods.	2911 rd.
6.	Reduce 133 in. to yards.	3 yd. 2 ft. 1 in.
7.	Reduce 181 in. to yards.	5 yd. 1 in.
8.	Reduce 2240 rd. to miles.	7 mi.
9.	Reduce 2200 rd. to miles.	6 mi. 280 rd.
10.	Reduce 1 mi. to yards.	1760 yd.
11.	Reduce 1 mi. to feet.	5280 ft.

SQUARE MEASURE

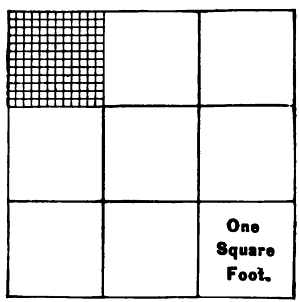
67. Square measure is used in measuring anything that has both length and breadth; that is, *two* dimensions.

A figure having 4 equal sides and 4 right angles is a **square.**

A *square inch* is a square, each side of which is 1 inch in length.

A *square foot* is a square, each side of which is 1 foot.

A *square yard* is a square, each side of which is 1 yard (3 feet).



Suppose the figure to represent a square yard. It will then be 3 feet each way, and contain 9 square feet. Each foot will be 12 inches each way, and contain 144 square inches.

The number of small squares in any large square is, therefore, equal to the number of units in one side multiplied by itself.

NOTE. — By 3 *feet square* is meant a square figure, each side of which is 3 feet, or 9 *square feet;* but by 3 *square feet* is meant 3 squares each one foot long and one foot wide; therefore, the difference in area between a figure 3 *feet square* and one containing 3 *square feet,* is 6 square feet.

TABLE

144	square inches (sq. in.)	= 1 square foot, marked sq. ft.
9	square feet	= 1 square yard, marked sq. yd.
$30\frac{1}{4}$	square yards	= 1 square rod, marked sq. rd.
160	square rods	= 1 acre, marked A.
640	acres	= 1 square mile, marked sq. mi.

NOTE. — The denominations most used in trade are the acre, the square yard, the square foot.

1. Reduce 8 sq. yd. to square inches. 10368 sq. in.
2. Reduce 4 A. to square rods. 640 sq. rd.
3. Reduce 1 sq. mi. to square rods. 102400 sq. rd.
4. Reduce 2 sq. yd. 3 sq. ft. to sq. in. 3024 sq. in.
5. Reduce 5 A. 100 sq. rd. to sq. rd. 900 sq. rd.
6. Reduce 960 sq. rd. to acres. 6 A.
7. Reduce 3888 sq. in. to square yards. 3 sq. yd.
8. Reduce 20000 sq. rd. to acres. 125 A.
9. Reduce 515280 sq. rd. to square miles. 5 sq. mi. 20 A. 80 sq. rd.
10. Reduce 4176 sq. in. to sq. yd. 3 sq. yd. 2 sq. ft.

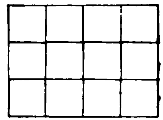
68. A **rectangle** is a figure having four sides and four right angles. See the figure on p. 99.

The **unit of measure** for surfaces is a *square* whose side is a linear unit, as a square inch, a square foot, etc.

The **area** or superficial contents of a figure is the number of times it contains its *unit of measure*.

1. How many square inches are there in a board 4 inches long and 3 inches wide?

EXPLANATION. — Dividing each of the longer sides into 4 equal parts, the shorter sides into 3 equal parts, and joining the opposite divisions by straight lines, the surface is divided into squares.



In each of the longer rows there are 4 squares, that is, as many as there are inches in the longer side; and there are as many such rows as there are inches in the shorter side. Hence,

The whole number of squares in the board is equal to the product obtained by multiplying together the numbers representing the length and breadth; that is, $4 \times 3 = 12$.

To find the area of a rectangle:

Rule. — *Multiply the number representing length by the number representing breadth; the product will be the number representing area.*

NOTE. — Both the length and breadth, if not in units of the same denomination, should be made so before multiplying.

2. In a floor 16 feet long and 12 feet wide, how many square feet are there? 192 sq. ft.

3. How many square yards of carpeting will cover a room 5 yards long and 4 yards wide? 20 sq. yd.

4. How many square yards are there in the floors of two rooms, one 18 feet long and 12 feet wide, the other 21 feet long and 15 feet wide? 59 sq. yd.

5. How many square yards are there in a ceiling 18 feet long and 14 feet wide? 28 sq. yd.

6. In a field 35 rods long and 32 rods wide, how many acres are there? 7 A.

7. How much will it cost to plaster a ceiling 21 feet long and 18 feet wide, at 17 cents per square yard? \$7.14.

69. Since the area of a rectangle is equal to the product of the length by the breadth, and since the product of two

numbers, divided by either of them, gives the other (§ **36**); therefore,

Rule. — *If the number representing the area of a rectangle is divided by the number representing either side, the quotient will be the number representing the other side.*

Illustration

In § **68**, Example 1, if the area 12 is divided by 4, the quotient 3 is the width; or, if 12 is divided by 3, the quotient 4 is the length.

NOTE. — Dividing the area of a rectangle by one of its sides, is really dividing the number of squares in the rectangle by the number of squares on one of its sides.

In dividing 12 by 4, the latter is not 4 linear inches, but the number of square inches in a rectangle 4 in. long and 1 in. wide. See the figure, § **68**.

1. A floor containing 132 square feet, is 11 feet wide. What is its length? 12 ft.

2. A floor is 18 feet long, and contains 30 square yards. What is its width? 15 ft.

3. A field containing 9 acres, is 45 rods in length. What is its width? 32 rd.

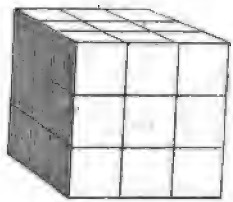
4. A field 35 rods wide, contains 21 acres. What is its length? 96 rd.

SOLID OR CUBIC MEASURE

70. Solid or cubic measure is used in measuring things having length, breadth, and thickness; that is, *three dimensions.*

A **cube** is a solid, having 6 equal faces, which are squares.

NOTE.—If each side of a cube is 1 inch long, it is called a cubic inch; if each side is 3 feet (1 yard) long, as represented in the figure, it is a cubic or solid yard.



The base of a cube, being 1 square yard, contains $3 \times 3 = 9$ square feet; and 1 foot high on this base, contains 9 solid feet; 2 feet high contains $9 \times 2 = 18$ solid feet; 3 feet high contains $9 \times 3 = 27$ solid feet. Also, it may be shown that 1 solid or *cubic* foot contains $12 \times 12 \times 12 = 1728$ solid or *cubic* inches.

The number of small cubes in any large cube is equal to the product of the numbers representing the length, breadth, and thickness.

NOTE.—Any solid, whose corners resemble a cube, is a rectangular solid; boxes and cellars are generally of this form.

The solid contents of a rectangular solid are found, as in the cube, by multiplying together the numbers representing length, breadth, and thickness.

TABLE

1728 cubic inches (cu. in.)	= 1 cubic foot, marked cu. ft.
27 cubic feet	= 1 cubic yard, marked cu. yd.
128 cubic feet = $8 \times 4 \times 4$ = 8 ft. long, 4 ft. wide, and 4 ft. high	= 1 cord, marked C.

NOTES.—1. A cord foot is 1 ft. in length of the pile which makes a cord. It is 4 feet wide, 4 feet high, and 1 foot long; hence, it contains 16 cubic feet, and 8 cord feet make 1 cord.

2. A perch of stone is a mass $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high, and contains $24\frac{3}{4}$ cu. ft.

3. The cord is used for dealing in firewood; the cubic yard, for excavations and fillings.

1. Reduce 2 cu. yd. to cubic inches. 93312 cu. in.

2. Reduce 28 cords of wood to cu. ft. 3584 cu. ft.

3. Reduce 34 cords of wood to cu. in. 7520256 cu. in.

4. Reduce 1 cord of wood to cu. in. 221184 cu. in.

5. Reduce 63936 cu. in. to cu. yd. 1 cu. yd. 10 cu. ft.

6. How many cubic feet are there in a rectangular solid 8 ft. long, 5 ft. wide, 4 ft. thick? 160 cu. ft.

7. How many cubic yards of excavation are required for a cellar 8 yd. long, 5 yd. wide, 2 yd. deep? 80 cu. yd.

8. How many cubic yards are there in a cellar, 18 feet long, 15 feet wide, 7 feet deep? 70 cu. yd.

9. In a pile of wood 40 feet long, 12 feet wide, and 8 feet high, how many cords are there? 30 C.

10. What will be the cost of a pile of wood 80 feet long, 8 feet high, and 4 feet thick, at $5.50 per cord? $110.

11. What will be the cost of excavating a cellar 24 ft. long, 15 ft. wide, and 6 ft. deep, at $1.25 per cubic yard or load? $100.

TIME MEASURE

71. Time measure is used in measuring time.

Table

	60 seconds (sec.)	= 1 minute, marked min.
	60 minutes	= 1 hour, marked hr.
	24 hours	= 1 day, marked da.
	365 days, 6 hours	= 1 year, marked yr.
	100 years	= 1 century, marked cen.
Also,	7 days	= 1 week, marked wk.
	4 weeks	= 1 month (nearly), marked mon.
	52 weeks	= 1 year, marked yr.
	12 calendar months	= 1 year, marked yr.
	365 days	= 1 common year.
	366 days	= 1 leap year.

NOTES.—1. The exact length of the mean solar or tropical year is 365 days, 5 hours, 48 minutes, 49.7 seconds.

To correct the error of considering 365 days as the length of the year, the following rule has been adopted:

Every year whose number is not divisible by 4 consists of 365 days.

Every year whose number is divisible by 100, but not by 400, consists of 365 days.

Every year, except the even centuries, whose number is divisible by 4, and the even centuries divisible by 400, consist of 366 days.

The year containing 366 days is called leap year, and the extra day is added to February, giving it 29 days instead of 28 days.

2. Among nearly all civilized nations the year is divided into 12 calendar months, numbered in their order as follows:

January,	1st month, 31 days.	July,	7th month, 31 days.
February,	2d month, 28 days.	August,	8th month, 31 days.
March,	3d month, 31 days.	September,	9th month, 30 days.
April,	4th month, 30 days.	October,	10th month, 31 days.
May,	5th month, 31 days.	November,	11th month, 30 days.
June,	6th month, 30 days.	December,	12th month, 31 days.

1. Reduce 2 hr. to seconds. 7200 sec.
2. Reduce 7 da. to minutes. 10080 min.
3. Reduce 1 da. 3 hr. 44 min. 3 sec. to seconds. 99843 sec.
4. Reduce 9 wk. 6 da. 10 hr. 40 min. to minutes. 100000 min.
5. Reduce 4 wk. 3 da. 4 min. to minutes. 44644 min.
6. Reduce 10800 seconds to hours. 3 hr.
7. Reduce 432000 seconds to days. 5 da.
8. Reduce 7322 seconds to hours. 2 hr. 2 min. 2 sec.
9. Reduce 4323 minutes to days. 3 da. 3 min.
10. Reduce 20280 minutes to weeks. 2 wk. 2 hr.
11. Reduce 41761 min. to months. 1 mo. 1 da. 1 min.

SPECIAL TABLES. — I. MEASURES OF WEIGHT

72. Troy weight is used in weighing gold, silver, and jewels.

24 grains (gr.)	= 1 pennyweight, marked pwt.
20 pennyweights	= 1 ounce, marked oz.
12 ounces	= 1 pound, marked lb.

NOTE. — The *standard unit* of all weight in the United States is the *Troy pound*, containing 5760 grains.

Apothecaries' weight is used only in *compounding* medicines.

20 grains (gr.)	= 1 scruple, marked sc. or ℈.
3 scruples	= 1 dram, marked dr. or ʒ.
8 drams	= 1 ounce, marked oz. or ℥.
12 ounces	= 1 pound, marked lb. or ℔.

The following are also used by apothecaries:

60 minims (or drops) ♏	= 1 fluid drachm, marked f. ʒ.
8 fluid drachms	= 1 fluid ounce, marked f. ℥.
16 fluid ounces	= 1 pt. (octarius), marked *O.*
8 pints	= 1 gal. (congius), marked *Cong.*

II. MEASURES OF LENGTH

The following measures are often mentioned, and most of them are still used in special professions:

12 lines	= 1 inch.	3 feet	= 1 pace.
3 barleycorns	= 1 inch.	6 feet	= 1 fathom.
4 inches	= 1 hand.	3 miles	= 1 league.
9 inches	= 1 span.	$69\frac{1}{6}$ miles (nearly)	= 1 degree.

NOTES. — 1. Surveyors use a chain four rods long, divided into 100 links of $7\frac{92}{100}$ inches each.

2. Engineers divide the foot into tenths and hundredths. The yard is divided similarly in estimating duties at the custom houses.

3. A degree is divided into 60 nautical or geographic miles. A nautical mile or *knot* is, therefore, nearly $1\frac{1}{6}$ common miles.

Circular measure is used in measuring circles.

60 seconds (″)	= 1 minute, marked ′.
60 minutes	= 1 degree, marked °.
360 degrees	= 1 circle.

NOTE. — The circumference is also divided into *quadrants* of 90° each, and into *signs* of 30° each.

III. MISCELLANEOUS TABLE

12 things	= 1 dozen, marked doz.
12 dozen	= 1 gross, marked gr.
12 gross	= 1 great gross.
20 things	= 1 score.
100 pounds of nails	= 1 keg.
196 pounds of flour	= 1 barrel.
200 pounds of pork or beef	= 1 barrel.
240 pounds of lime	= 1 cask.
24 sheets of paper	= 1 quire.
20 quires	= 1 ream.
2 reams	= 1 bundle.

Examples in Special Tables

73. 1. Reduce 5 lb. 4 oz. Troy to ounces. 64.

2. Reduce 8 lb. 9 oz. 13 pwt. 17 gr. to grains. 50729 gr.

3. Reduce 12530 gr. to pounds. 2 lb. 2 oz. 2 pwt. 2 gr.

4. Reduce 4 lb. 5 oz. 2 gr. to grains. 25442 gr.

5. Reduce 975 sc. to pounds. 3 lb. 4 oz. 5 dr.

6. What is the height of a horse of $16\frac{1}{2}$ hands? 5 ft. 6 in.

7. A field measures 24 chains in length and 15 chains in breadth. How many acres are there in it? 36 A.

8. A cistern contains 267 cubic feet 624 cubic inches. How many gallons does it hold? (§ 64) 2000 gal.

9. Reduce 8° 41′ 45″ to seconds. 31305″.

10. Reduce 61° 59′ 28″ to seconds. 223168″.

11. Reduce 915′ to degrees. 15° 15′.

12. Reduce 3661″ to degrees. 1° 1′ 1″

13. Find the cost of 6 gross of screws at 5 cents a dozen. $3.60.

14. A man is 4 score and 10. How old is he? 90 yr.

15. At 18 cents a quire, how much will 3 bundles of paper cost? $21.60.

Miscellaneous Examples

74. 1. Find the cost of 2 bu. of plums at 5¢ a pint. $6.40.

2. Find the cost of 3 bu. 2 pk. of peaches at 50¢ a peck? $7.

3. Find the cost of 5 bu. 2 pk. of barley at 50¢ a bushel. $2.75.

4. At 15¢ a peck, how many bushels of potatoes can be bought for $3? 5 bu.

5. At 2¢ a pint, how much milk can be bought with $1.66? 10 gal. 1 qt. 1 pt.

6. I put 91 bu. of wheat into bags containing 3 bu. 2 pk. each. How many bags were required? 26.

NOTE. — Reduce both quantities to pecks, and then divide.

7. How many spikes, weighing 4 oz. each, are in a parcel weighing 15 lb. 12 oz.? 63.

8. I bought 44 cwt. 52 lb. of cheese; each cheese weighed 9 lb. 15 oz. How many cheeses did I buy? 448.

9. How many kegs, of 84 lb. each, can be filled from a hogshead of sugar weighing 14 cwt. 28 lb.? 17.

10. How many boxes, containing 12 lb. each, can be filled from 7 cwt. 56 lb. of tobacco? 63.

11. If a family use 3 lb. 13 oz. of sugar a week, how long will 6 cwt. 10 lb. last them? 160 wk.

12. How much will 2 acres 125 square rods of land cost at 20 cents a square rod? $89.

13. A farmer has a field of 16 A. 53 sq. rd. to divide into lots of 1 A. 41 sq. rd. each. How many lots will it make? 13.

14. How many cubic inches are there in a block of marble 2 ft. long, 2 ft. high, 2 ft. wide? 13824 cu. in.

15. One cubic foot of water weighs 1000 oz. avoirdupois. How much do 5 cu. ft. weigh? 312 lb. 8 oz.

16. What is the weight of a quantity of water occupying the space of 1 cord of wood, each cubic foot of water weighing 1000 ounces avoirdupois? 4 T.

17. A cubic foot of oak weighs 950 oz. avoirdupois. How much do 2 cords of oak weigh? 7 T. 12 cwt.

18. Find the cost of 63 gallons of maple sirup, at 20 cents a pint. $100.80.

19. Find the cost of 5 barrels of molasses, each containing 31 gal. 2 qt., at 10 cents a quart. $63.

20. At 5 cents a pint, what quantity of molasses can be bought for $2? 5 gal.

21. How many dozen bottles, each bottle holding 3 qt. 1 pt., can be filled from 63 gal. of cider? 6 doz.

22. How many kegs, of 4 gal. 3 qt. 1 pt. each, can be filled from 58 gal. 2 qt.? 12.

23. If a human heart beats 70 times a minute, how many times will it beat in a day? 100800.

24. How many seconds are there in the month of February, 1904? 2505600 sec.

25. If a ships sails 8 miles an hour, how many miles will it sail in 3 wk. 2 da. 3 hr.? 4440 mi.

26. A horse is fed 1 peck of oats daily. If oats cost 44 cents a bushel, how much will it cost to feed him a year of 365 days? $40.15.

27. A flour dealer bought 40 barrels of extra fine Minnehaha flour for 3¢ a pound, and sold it for 5¢ a pound. How much did he gain? $156.80.

ADDITION OF COMPOUND DENOMINATE NUMBERS

75. The processes of adding, subtracting, multiplying, and dividing compound denominate numbers are governed by the same principles as in simple numbers.

1. A farmer sold three lots of wheat. The first lot contained 25 bu. 3 pk.; the second, 14 bu. 2 pk.; the third, 32 bu. 1 pk. How much did he sell?

SOLUTION. — Place units of the same denomination in the same column (§ **17**). Beginning with pecks, and adding, the sum is 6, which is reduced to bushels by dividing by 4, the number of pecks in a bushel. There being 2 pecks left, write the 2 under the column of pecks; adding the 1 bushel to the column of bushels, the sum is 72, which write under the column of bushels.

OPERATION

bu.	pk.
25	3
14	2
32	1
72	2

2.

bu.	pk.	qt.	pt.
3	2	0	1
4	0	6	1
1	3	7	1
9	2	6	1

3.

bu.	pk.	qt.	pt.
7	3	7	1
6	2	0	0
9	2	4	1
24	0	4	0

Rule. — 1. *Write the numbers to be added, placing units of the same denomination in the same column.*

2. *Begin with the lowest denomination, add the numbers, and divide their sum by the number of units of this denomination which make a unit of the next higher.*

3. *Write the remainder under the column added, and add the quotient to the next column.*

4. *Proceed in the same manner with all the columns to the last, under which write its entire sum.*

Notes. — 1. In writing compound denominate numbers, if any intermediate denomination is wanting, supply its place with a cipher.

2. In adding simple numbers we carry one for every ten, because ten units of a lower order always make one of the next higher; but, in compound denominate numbers, the scale varies, and we carry one for the number of the lower order, which makes one of the next higher.

Add:

4.

bu.	pk.	qt.	pt.
8	1	7	1
7	3	2	1
9	2	7	1

[illegible] - 0 - 1 - 1

5.

gal.	qt.	pt.	gi.
40	3	1	3
16	1	0	2
71	2	1	2

1[illegible]8 - 3 - 1 - 3

6.

T.	cwt.	lb.	oz.
45	3	53	10
14	14	75	15
19	17	18	13

79 - 15 - 4[illegible] - 6

7.

mi.	rd.
28	129
64	280
17	275

111 - 44

8.

yd.	ft.	in.
4	2	11
3	1	9
5	1	8

9.

A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
41	51	13	8	115
64	104	5	7	109
193	155	4	5	137

10.

da.	hr.	min.
16	18	28
13	15	49
19	16	53

11.

cu. yd.	cu. ft.	cu. in.
50	18	900
45	17	828
46	20	990

12.

mo.	wk.	da.	hr.	min.	sec.
3	0	0	23	51	40
1	2	4	19	30	37
3	1	5	13	27	18

13. Five loads of wheat measured thus: 21 bu. 3 pk.; 14 bu. 1 pk.; 23 bu. 2 pk.; 18 bu. 1 pk.; 22 bu. 1 pk. How many bushels were there in all? 100 bu.

14. Add 13 lb. 11 oz.; 17 lb. 13 oz.; 14 lb. 14 oz.; 16 lb.; 19 lb. 7 oz.; and 17 lb. 9 oz. 99 lb. 6 oz.

15. Two men depart from the same place. One travels 104 mi. 50 rd. due east; the other, 95 mi. 270 rd. due west. How far apart are they? 200 mi.

16. A man has one farm of 186 A. 134 sq. rd.; another of 286 A. 17 sq. rd.; a third of 113 A. 89 sq. rd. How much is there in all? 586 A. 80 sq. rd.

17. Add 17 sq. yd. 3 sq. ft. 119 sq. in.; 18 sq. yd. 141 sq. in.; 23 sq. yd. 7 sq. ft.; 29 sq. yd. 5 sq. ft. 116 sq. in. 88 sq. yd. 8 sq. ft. 88 sq. in.

18. A has 4 piles of wood. In the first there are 7 C. 78 cu. ft.; in the second, 16 C. 24 cu. ft.; in the third, 35 C. 127 cu. ft.; in the fourth, 29 C. 10 cu. ft. How much is there in all? 88 C. 111 cu. ft.

19. A farmer raised 200 bu. 3 pk. of oats, 143 bu. 1 pk. of barley, 400 bu. 3 pk. of corn, 255 bu. 1 pk. of wheat. How much did he raise altogether? 1000 bu.

20. I sold to A 4642 gal. 3 qt. 1 pt. of oil; to B, 945 gal.; to C, 1707 gal. 1 pt.; to D, 10,206 gal. 1 qt. How many hogsheads of 63 gallons each did I sell?

277 hogsheads 50 gal. 1 qt.

SUBTRACTION OF COMPOUND DENOMINATE NUMBERS

76. **1.** I have 67 bu. 2 pk. of wheat. How much will remain after 34 bu. 3 pk. are sold?

SOLUTION. — Write the smaller number under the greater, placing units of the same denomination in the same column. 3 pk. cannot be taken from 2 pk. Take 1 bu. from 67 bu., reduce it to 4 pecks, and add the 2 pk., making 6 pk.; 3 pk. from 6 pk. leaves 3 pk.; 34 bu. from 66 bu. leaves 32 bu. The difference is, therefore, 32 bu. 3 pk.

OPERATION

bu.	pk.
67	2
34	3
32	3

NOTE. — Instead of diminishing the 67 bu. by 1, the result will be the same by increasing the lower number 34 bu. by 1, as is done in subtraction of simple numbers.

2.

	bu.	pk.	qt.	pt.
From	12	0	1	0
Take	8	2	1	1
	3	1	7	1

3.

	bu.	pk.	qt.	pt.
From	5	0	0	0
Take	1	0	0	1
	3	3	7	1

Rule. — 1. *Write the smaller number under the greater, placing units of the same denomination in the same column.*

2. *Begin with the lowest denomination, and, if possible, take the lower number from the one above it.*

3. *But, if the lower number of any denomination is greater than the upper, increase the upper number by as many units of that denomination as make one of the next higher; subtract as before, and diminish by one the upper number of the next higher denomination.*

4. *Proceed in the same manner with each denomination.*

Subtract:

4.

gal.	qt.	pt.	gi.
43	1	1	2
23	3	1	3

5.

T.	cwt.	lb.	oz.
16	7	18	14
5	6	75	15

6.

mi.	rd.
18	198
11	236

7.

yd.	ft.	in.
4	1	10
2	1	11

8.

sq. yd.	sq. ft.	sq. in.
19	6	72
16	6	112

9.

C.	cu. ft.
28	116
19	119

10.

da.	hr.	min.	sec.
245	17	40	37
190	11	44	42

11. If 2 bu. 1 pk. 1 qt. are taken from a bag containing 4 bushels of hickory nuts, what quantity will remain?
1 bu. 2 pk. 7 qt.

12. I bought 46 lb. 4 oz. of rice. After selling 19 lb. 8 oz., how much remained? 26 lb. 12 oz.

13. A wagon loaded with hay weighs 32 cwt. 66 lb.; the wagon alone weighs 8 cwt. 67 lb. What is the weight of the hay? 23 cwt. 99 lb.

14. It is 24899 miles round the earth. After a man who is traveling around it has traveled 100 mi. 41 rd., what distance remains for him to travel? 24798 mi. 279 rd.

15. I had a farm containing 146 A. 80 sq. rd. of land. I gave my son 86 A. 94 sq. rd. How much was left?
59 A. 146 sq. rd.

16. A cask containing 63 gal. leaked; only 51 gal. 1 qt. 2 gi. remained. How much was lost?
11 gal. 2 qt. 1 pt. 2 gi.

17. From 5 da. 10 hr. 27 min. 15 sec. take 2 da. 4 hr. 13 min. 29 sec. 3 da. 6 hr. 13 min. 46 sec.

77. In finding the time between any two dates, consider 30 days as 1 month, and 12 months as 1 year.

1. A note, dated April 14, 1900, was paid February 12, 1902. Find the time between these dates.

SOLUTION.—In writing the dates, observe that February is the 2d month of the year and April the 4th; then, from 1902 yr. 2 mo. 12 da. subtract 1900 yr. 4 mo. 14 da. The remainder is 1 yr. 9 mo. 28 da.

OPERATION

yr.	mo.	da.
1902	2	12
1900	4	14
1	9	28

2. The Independence of the United States was declared July 4, 1776. What length of time had elapsed on the 1st of September, 1903? 127 yr. 1 mo. 27 da.

3. The first crusade ended July 15, 1099; the third crusade, July 12, 1192. Find the difference of time between these dates. 92 yr. 11 mo. 27 da.

4. Magna Charta was signed June 15, 1215; Mary, Queen of Scots, was beheaded February 8, 1587. Find the difference of time between these dates.

371 yr. 7 mo. 23 da.

5. The battle of Hastings was fought Oct. 14, 1066; William, Prince of Orange, landed at Torbay Nov. 5, 1688. What was the difference of time between the two events? 622 yr. 21 da.

6. The battle of Austerlitz was fought December 2, 1805; the battle of Waterloo, June 18, 1815. Find the difference of time. 9 yr. 6 mo. 16 da.

78. To find the time between two dates in days:

1. Find the number of days from May 10 to October 21.

SOLUTION.—Of May, there remain 31 − 10 = 21 days; there are 30 days in June, 31 in July, 31 in August, 30 in September, and 21 in October; then the number of days from May 10 to October 21, is 21 + 30 + 31 + 31 + 30 + 21 = 164.

OPERATION

	31
	10
May,	21
June,	30
July,	31
Aug.,	31
Sept.,	30
Oct.,	21
	164 da.

2. Find the number of days from March 17 to September 12. 179 da.

3. A note dated April 18, 1903, is due June 20, 1903. How many days does it run? 63 da.

4. A note dated Sept. 5, 1903, is due Dec. 7, 1903. How many days does it run? 93 da.

5. Find the number of days from Oct. 12, 1900, to May 25, 1901. 225 da.

6. Find the number of days from Aug. 20, 1903, to March 8, 1904. 201 da.

MULTIPLICATION OF COMPOUND DENOMINATE NUMBERS

79. 1. A farmer takes to mill 5 bags of wheat, each containing 2 bu. 3 pk. How much has he in all?

SOLUTION. — Begin at the lowest denomination for convenience. Multiply the 3 pk. by 5, making 15 pk., which reduced, gives 3 bu. and 3 pk.; write the 3 pk. under the pecks. Then, multiply the 2 bu. by 5, add to the product the 3 bu., and write the 13 bu. under the bushels.

OPERATION

bu.	pk.
2	3
	5
13	3

Rule. — 1. *Write the multiplier under the lowest denomination of the multiplicand.*

2. *Multiply the lowest denomination first, and divide the product by the number of units of this denomination which make a unit of the next higher, write the remainder under the denomination multiplied, and add the quotient to the product of the next higher denomination.*

3. *Proceed in like manner with all the denominations, writing the entire product at the last.*

NOTE. — There are *two* differences between multiplication of simple and of compound denominate numbers. 1. In simple numbers it is more convenient to use *one* figure of the multiplier at a time; in com-

pound numbers it is better to use the *entire multiplier* each time. 2. In simple numbers the scale is *uniform;* in compound numbers it *varies with the table.*

2. Multiply 2 bu. 1 pk. 1 qt. 1 pt. by 6.
13 bu. 3 pk. 1 qt.

3. Find the weight of 9 hogsheads of sugar, each weighing 8 cwt. 62 lb. 3 T. 17 cwt. 58 lb.

4. If a ship sails 208 mi. 176 rd. a day, how far will it sail in 15 days? 3128 mi. 80 rd.

5. Multiply 23 cu. yd. 9 cu. ft. 228 cu. in. by 12.
280 cu. yd. 1 cu. ft. 1008 cu. in.

6. Multiply 16 cwt. 74 lb. by 119. 99 T. 12 cwt. 6 lb.

7. Multiply 47 gal. 3 qt. 1 pt. by 59.
2824 gal. 2 qt. 1 pt.

8. A travels 27 mi. 155 rd. in 1 day, how far will he travel in one month of 31 days? 852 mi. 5 rd.

9. In 17 piles of wood, each pile containing 7 C. 98 cu. ft., how much wood is there? 132 C. 2 cu. ft.

10. Multiply 2 wk. 4 da. 13 hr. 48 min. 39 sec. by 75.
49 mo. 3 wk. 3 hr. 48 min. 45 sec.

DIVISION OF COMPOUND DENOMINATE NUMBERS

80. The *divisor* may be either a simple or a compound number. This gives rise to two cases:

I. To find how often one compound number is contained in another compound number:

This is done by reducing both divisor and dividend to the same denomination before dividing (§ **74**, Ex. 6 and 8).

II. To divide a compound denominate number into a given number of equal parts:

This is properly division of compound denominate numbers.

1. Divide 14 bu. 2 pk. 1 qt. by 3.

OPERATION

	bu.	pk.	qt.
3)	14	2	1
	4	3	3

Solution. — Divide the highest denomination first, so that, if there is a remainder, it may be reduced to the next lower denomination, and added to it. 3 in 14 is contained 4 times, and 2 bu. are left; write the 4 under the bushels, and reduce the remaining 2 bu. to pk., to which add the 2 pk., making 10 pk. This, divided by 3, gives a quotient of 3 pk., with 1 pk. remaining; which, reduced to qt., with 1 qt. added, gives 9 qt. This, divided by 3, gives a quotient 3, which is written under the quarts.

Divide:

2.

	bu.	pk.	qt.
7)	33	2	6
	4	3	2

3.

	da.	hr.	min.	sec.
5)	17	12	56	15
	3	12	11	15

Rule. — 1. *Write the quantity to be divided in the order of its denominations, beginning with the highest; place the divisor on the left.*

2. *Begin with the highest denomination, divide each number separately, and write the quotient beneath.*

3. *If a remainder occurs after any division, reduce it to the next lower denomination, and, before dividing, add to it the number of its denomination.*

Note. — Each *partial* quotient is of the same denomination as that part of the dividend from which it is derived.

4. Divide 69 A. 64 sq. rd. by 16. 4 A. 54 sq. rd.

5. 265 lb. 10 oz. ÷ 50. 5 lb. 5 oz.

6. 45 T. 18 cwt. ÷ 17. 2 T. 14 cwt.

7. 114 da. 22 hr. 45 min. 18 sec. ÷ 54.
2 da. 3 hr. 5 min. 17 sec.

8. 309 bu. 2 pk. 2 qt. ÷ 78. 3 bu. 3 pk. 7 qt.

9. 127 gal. 3 qt. 1 pt. 3 gi. ÷ 63. 2 gal. 1 gi.

10. 788 mi. 169 rd. ÷ 319. 2 mi. 151 rd.

11. Add 35 lb. 9 oz., 75 lb. 14 oz., 85 lb. 15 oz.; from the sum take 186 lb. 14 oz.; multiply the remainder by 8; divide the product by 64. What is the result? 1 lb. 5 oz.

LONGITUDE AND TIME

81. Difference of longitude and time between different places:

The circumference of the earth, like other circles, is divided into 360 equal parts, called *degrees of longitude.*

The sun appears to pass entirely round the earth, 360°, once in 24 hours, *one day;* and in one hour it passes over 15°. $(360° \div 24 = 15°.)$

As 15° equal 900′, and 1 hour equals 60 minutes of *time*, therefore, the sun in 1 minute of *time* passes over 15′ of longitude. $(900' \div 60 = 15'.)$

As 15′ equal 900″, and 1 minute of *time* equals 60 seconds of *time*, therefore, in 1 second of *time* the sun passes over 15″ of longitude. $(900'' \div 60 = 15''.)$

Table for Comparing Longitude and Time

Two Places Distant from Each Other

15° of longitude differ 1 hour in time.
15′ of longitude differ 1 min. in time.
15″ of longitude differ 1 sec. in time.

I. To find the time corresponding to any difference of longitude:

Rule. — *Divide the longitude by* 15, *according to the rule for division of compound denominate numbers, and mark the quotient* hr. min. sec., *instead of* ° ′ ″.

II. To find the longitude corresponding to any difference of time:

Rule. — *Multiply the time by* 15, *according to the rule for multiplication of compound denominate numbers, and mark the product* ° ′ ″ *instead of* hr. min. sec.

1. The difference of longitude between two places is 30°. What is the difference of time? 2 hr.

2. The difference of longitude between two places is 71° 4′. What is the difference of time? 4 hr. 44 min. 16 sec.

3. The difference of longitude between New York and Cincinnati is 10° 35′. What is the difference of time? 42 min. 20 sec.

4. The difference of time between Cincinnati and Philadelphia is 37 min. 20 sec. What is the difference of longitude? 9° 20′.

5. The difference of time between New York and St. Louis is 1 hr. 4 min. 56 sec. What is the difference of longitude? 16° 14′.

6. The difference of time between London and Washington is 5 hr. 8 min. 4 sec. What is the difference of longitude? 77° 1′.

Difference in Time

82. It is noon (12 o'clock) at any place when the sun is on the meridian of that place.

As the sun appears to travel from the east *toward* the west, when it is noon at any place it is *after* noon *east* of that place, and *before* noon *west* of that place.

Hence, a place has *later* or *earlier time* than another, according as it is *east* or *west* of it. Therefore,

When the time at one place is given, the time at another, if **east** *of this, is found by* **adding** *their difference of time; if* **west**, *by* **subtracting** *their difference of time.*

7. When it is noon at Cincinnati, what is the time at Philadelphia? 37 min. 20 sec. past noon.

8. When it is 11 o'clock A.M. at New York, what is the time in longitude 30° east of New York? 1 P.M.

9. When it is 12 o'clock (noon) at Philadelphia, what is the time at Cincinnati? 11 hr. 22 min. 40 sec. A.M.

10. When it is 11 o'clock A.M. at New York, what is the time at St. Louis? 9 hr. 55 min. 4 sec. A.M.

NOTE. — The **standard time system**, adopted in 1883 by railroads and cities of the United States, makes the answers to the last four problems merely theoretical. Standard time is the clock time of selected meridians 15° apart; viz., the meridians 75°, 90°, 105°, and 120° west from Greenwich. **Eastern standard time**, the clock time of the meridian 75° west of Greenwich, is 5 hours slower than Greenwich time. **Central standard time**, the clock time of 90° west of Greenwich, is just 1 hour slower than Eastern time. **Mountain standard time**, the clock time of the meridian of 105°, is just 1 hour slower than Central time. **Pacific standard time**, the clock time of the meridian of 120°, is just 1 hour slower than Mountain time. Places within 7½° east or west of a meridian have the time of that meridian. Therefore, *the time between two places differs only by whole hours.*

11. What is the difference in standard time between Boston and Chicago?

Since Boston has Eastern standard time (75°) and Chicago Central standard time (90°), the difference in standard time is just 1 hour.

12. What is the difference in standard time between Philadelphia and Cincinnati? 1 hour.

13. When it is 11 o'clock A.M. at New York, what is the standard time at St. Louis? 10 A.M.

FACTORING

83. A number that expresses whole units is called an **integer** or an **integral number.**

Thus, 1, 2, 50, etc., are integral numbers.

Factors of a number are two or more integral numbers, the product of which equals the given number (§ **28**).

Thus, 2 and 3 are factors of 6, because $2 \times 3 = 6$; 2, 3, and 5 are factors of 30, because $2 \times 3 \times 5 = 30$.

Note.—A number may be the product of more than one set of factors. Thus, $2 \times 6 = 12$, $3 \times 4 = 12$, and $2 \times 2 \times 3 = 12$.

A **multiple** of a number is a product of which the number is a factor.

Thus, 6 is a multiple of 3; 30 is a multiple of 5.

An **even number** is a number that is exactly divisible by 2; an **odd number** is a number that is not exactly divisible by 2.

Thus, 2, 4, 6, 8 are even numbers and 1, 3, 5, 7 are odd numbers.

A **prime number** has no factors except itself and 1.

Thus, 5, 11, 17 are prime numbers.

A **composite number** has two or more factors beside itself and 1.

Thus, 6, 12, 30 are composite numbers.

A **prime factor** is a factor which is a prime number.

Thus, 3 is a prime factor of 12.

A **factor** is **common** to two or more numbers when it is a factor of each of them.

Thus, 3 is a common factor of 12 and 15.

NOTE. — Sometimes the smallest of two or more numbers may be the common factor. Thus, 6 is a common factor of 6, 12, and 18.

Two or more numbers are *prime to each other* when they have no common factor.

Thus, 9 and 10 are prime to each other.

A **common divisor** (C. D.) of two or more numbers is any common factor.

Thus, 2, 3, and 6 are each a common divisor of 12 and 18.

The **greatest common divisor** (G. C. D.) of two or more numbers is the greatest common factor.

Thus, 6 is the greatest common divisor of 12 and 18.

A **common multiple** (C. M.) of two or more numbers is any multiple of all of them.

Thus, 6, 12, 18, etc., are common multiples of 2 and 3.

The **least common multiple** (L. C. M.) of two or more numbers is the least multiple of all of them.

Thus, 6 is the least common multiple of 2 and 3.

Factoring is the process of resolving composite numbers into their factors.

84. **To find the prime numbers:**

NOTE. — All the prime numbers except 2 are odd numbers.

Rule. — 1. *Write the odd numbers in a series* 1, 3, 5, 7, 9, *etc.*

2. *After* 3 *erase every* 3*d number; after* 5 *erase every* 5*th number; after* 7 *erase every* 7*th number; after* 11 *erase every* 11*th number, etc.*

3. *Then 2 and the numbers that remain are the prime numbers.*

EXERCISE. — Find the prime numbers from 1 to 100.

85. The operations of factoring depend upon the following principles:

Principles. — I. *A factor of a number exactly divides it.*

Thus, 5 is a factor of 30 and is contained in it 6 times.

II. *A multiple of a number exactly contains it.*

Thus, 30 is a multiple of 5 and contains it 6 times.

III. *A factor of a number is a factor of any multiple of that number.*

Thus, 3 being a factor of 6 is a factor of 12, 18, 24, etc.

IV. *A composite number is equal to the product of all its prime factors.*

Thus, the prime factors of 30 are 2, 3, and 5; $2 \times 3 \times 5 = 30$.

86. In resolving numbers into their prime factors it will be found convenient to remember the following facts:

1. *Two* is a factor of every even number.

Thus, 2 is a factor of 4, 6, 8, 10, etc.

2. *Three* is a factor of a number when the sum of its digits is divisible by 3.

Thus, 3 is a factor of 2457; for $2 + 4 + 5 + 7 = 18$, which is divisible by 3.

3. *Four* is a factor of a number when the number expressed by its two right-hand figures is divisible by 4, or when these figures are ciphers.

Thus, 4 is a factor of 316 because 16 is divisible by 4; 4 is a factor of 1200 because the two right-hand figures of 1200 are ciphers.

4. *Five* is a factor of every number whose unit figure is 0 or 5.

Thus, 5 is a factor of 10, 15, 20, 25, etc.

5. *Six* is a factor of every even number when the sum of its digits is divisible by 3.

Thus, 6 is a factor of 336 because 336 is an even number and $3 + 3 + 6$ is divisible by 3.

6. *Eight* is a factor of a number when its three right-hand figures are divisible by 8, or when they are ciphers.

Thus, 8 is a factor of 6848 because 848 is divisible by 8; 8 is a factor of 12000 because the three right-hand figures are ciphers.

7. *Nine* is an exact divisor of a number when the sum of its digits is divisible by 9.

Thus, 9 is an exact divisor of 927 because $9 + 2 + 7$ is divisible by 9.

NOTE.—Whether the prime numbers 7, 11, 13, etc., are factors of a number or not is best ascertained by trial.

87. To resolve a number into its prime factors:

1. Resolve 30 into its prime factors.

SOLUTION.—2 is a factor of 30 (§ **86**). Dividing 30 by 2, the quotient is 15. 3 being a factor of 15 (§ **86**) is also a factor of 30 (§ **85**, Prin. 3). Dividing 15 by 3 the quotient is 5, a prime number. Then, 2, 3, and 5 are the prime factors of 30.

OPERATION

$$\begin{array}{r|l} 2 & 30 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Rule. — 1. *Divide the given number by any prime number that will exactly divide it.*

2. *Divide the quotient in the same manner; and so continue to divide, until a quotient is obtained which is a prime number.*

3. *The several divisors and the last quotient will be the prime factors of the given number.*

NOTE. — It will be most convenient to divide each time by the *smallest* prime number.

Resolve the following into their prime factors:

2.	4.	2, 2.	23.	39.	3, 13.
3.	8.	2, 2, 2.	24.	40.	2, 2, 2, 5.
4.	9.	3, 3.	25.	42.	2, 3, 7.
5.	10.	2, 5.	26.	44.	2, 2, 11.
6.	12.	2, 2, 3.	27.	45.	3, 3, 5.
7.	14.	2, 7.	28.	46.	2, 23.
8.	15.	3, 5.	29.	48.	2, 2, 2, 2, 3.
9.	16.	2, 2, 2, 2.	30.	49.	7, 7.
10.	18.	2, 3, 3.	31.	50.	2, 5, 5.
11.	20.	2, 2, 5.	32.	70.	2, 5, 7.
12.	22.	2, 11.	33.	77.	7, 11.
13.	24.	2, 2, 2, 3.	34.	91.	7, 13.
14.	25.	5, 5.	35.	105.	3, 5, 7.
15.	26.	2, 13.	36.	119.	7, 17.
16.	27.	3, 3, 3.	37.	133.	7, 19.
17.	28.	2, 2, 7.	38.	154.	2, 7, 11.
18.	32.	2, 2, 2, 2, 2.	39.	210.	2, 3, 5, 7.
19.	34.	2, 17.	40.	231.	3, 7, 11.
20.	35.	5, 7.	41.	330.	2, 3, 5, 11.
21.	36.	2, 2, 3, 3.	42.	462.	2, 3, 7, 11.
22.	38.	2, 19.	43.	2310.	2, 3, 5, 7, 11.

88. To find the prime factors common to two or more numbers:

1. What prime factors are common to 30 and 42?

SOLUTION. — Write the numbers in a line. 2 is a prime factor of both 30 and 42 (§ **86**). Dividing by 2, the quotients are 15 and 21. 3 is a prime factor of both 15 and 21 (§ **86**); and consequently of both 30 and 42 (§ **85**, Prin. 3). Dividing by 3, the quotients 5 and 7 are prime to each other (§ **83**). Then 2 and 3 are the common factors.

OPERATION

$$\begin{array}{r|rr} 2 & 30 & 42 \\ \hline 3 & 15 & 21 \\ \hline & 5 & 7 \end{array}$$

Rule. — 1. *Write the given numbers in a line.*

2. *Divide by any prime number that will exactly divide all of them; divide the quotients in the same manner; and so continue to divide until two or more of the quotients are prime to each other.*

3. *Then the several divisors will be the common factors.*

What prime factors are common to:

2.	60 and 90?	2, 3, 5.
3.	56 and 88?	2, 2, 2.
4.	72 and 84?	2, 2, 3.
5.	54 and 90?	2, 3, 3.
6.	81 and 108?	3, 3, 3.
7.	80 and 100?	2, 2, 5.
8.	84 and 126?	2, 3, 7.
9.	52, 68, and 76?	2, 2.
10.	66, 78, and 102?	2, 3.
11.	63, 99, and 117?	3, 3.
12.	50, 70, and 110?	2, 5.

13. 45, 75, and 105? **3, 5.**

14. 75, 125, and 175? **5, 5.**

15. 42, 70, and 98? **2, 7.**

16. 33, 55, 77, and 121? **11.**

17. 39, 65, 91, and 104? **13.**

18. 34, 51, 85, and 102? **17.**

19. 38, 57, 95, and 114? **19.**

GREATEST COMMON DIVISOR

89. Finding the G. C. D. of two or more numbers depends upon the following principle:

Principle. — *The G. C. D. of two or more numbers contains all the prime factors common to the numbers, and no other factor.*

Thus, the G. C. D. of 12 and 18 is 6; it contains the common prime factors 2 and 3; it must contain both of them, else it would not be the *greatest* C. D.; it can contain no other factor, else it would not divide both 12 and 18.

1. Find the G. C. D. of 30 and 42.

First Method

OPERATION

2	30	42
3	15	21
	5	7

SOLUTION. — The prime factors common to 30 and 42 are 2 and 3 (§ **88**); their product is 6; then the G. C. D. of 30 and 42 is 6 (Prin.).

Rule. — 1. *Find the prime factors common to the given numbers.*

2. *Their product will be the greatest common divisor.*

Second Method

SOLUTION. — Dividing 42 by 30, the remainder is 12; dividing 30 by 12, the remainder is 6; dividing 12 by 6, the remainder is 0. Then 6 is the G. C. D. of 30 and 42. For, $30 = 6 \times 5$ and $42 = 6 \times 7$; then, because 5 and 7 are prime to each other, 6 must contain all the prime factors common to 30 and 42; it is, therefore, their G. C. D. (Prin.).

OPERATION

```
30)42(1
   30
   --
   12)30(2
      24
      --
       6)12(2
         12
         --
```

Rule. — 1. *Divide the greater number by the less, the divisor by the remainder, and so on, always dividing the last divisor by the last remainder, until nothing remains.*

2. *The last divisor will be the greatest common divisor.*

NOTE. — To find the G. C. D. of more than two numbers, first find the G. C. D. of two of them, then of that common divisor and one of the remaining numbers, and so on for all the numbers; the last common divisor will be the G. C. D. of all the numbers.

Find the greatest common divisor of the following numbers:

2.	16, 24, and 40.	8.
3.	24, 36, and 60.	12.
4.	36, 54, and 90.	18.
5.	40, 60, and 100.	20.
6.	54, 81, and 108.	27.
7.	60, 90, and 120.	30.
8.	32, 48, 80, and 112.	16.
9.	48, 72, 96, and 120.	24.
10.	72, 108, 144, and 180.	36.
11.	62 and 93.	31.

12. 78 and 130. **26.**
13. 161 and 253. 23.
14. 247 and 323. 19.
15. 391 and 697. 17.
16. 2145 and 3471. 39.
17. 16571 and 38363. 227.
18. 72, 120, and 132. 12.
19. 75, 125, and 165. 5.

LEAST COMMON MULTIPLE

90. Finding the L. C. M. of two or more numbers depends upon the following principle:

Principle. — *The L. C. M. of two or more numbers contains all the prime factors of each number and no other factor.*

Thus, the L. C. M. of 12 and 18 is 36; its prime factors are 2, 2, 3, and 3; it must contain all these factors, else it would not contain both the numbers; it must contain no other factor, else it would not be the *least common multiple.*

1. Find the L. C. M. of 5, 9, 12, and 30.

SOLUTION. — The prime factors of 5 are 1 and 5; those of 9 are 3 and 3; of 12, 2, 2, and 3; and of 30, 2, 3, and 5. Then, the prime factors of the L. C. M. are 5, 3, 3, 2, 2, and no other factor (Prin.). Hence, 180 is the L. C. M.

OPERATION

$$5 = 1 \times 5$$
$$9 = 3 \times 3$$
$$12 = 2 \times 2 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$5 \times 3 \times 3 \times 2 \times 2 = 180$$

The process of factoring and selecting the prime factors for the L. C. M. is very much simplified by the operation in the form of Short Division, as here shown. Since a multiple of a number is also a multiple of all its factors, 5, which is a factor of 30, may be disregarded.

OPERATION

$$\begin{array}{r|rrr} 2 & 9 & 12 & 30 \\ \hline 3 & 9 & 6 & 15 \\ \hline & 3 & 2 & 5 \end{array}$$

$$2 \times 3 \times 3 \times 2 \times 5 = 180$$

Rule. — 1. *Write the given numbers in a line, disregarding any that are factors of another.*

2. *Divide by any prime number that will exactly divide two or more of them.*

3. *Write the quotients and undivided numbers in a line beneath.*

4. *Divide these numbers in the same manner, and so continue the operation until a line is reached in which the numbers are all prime to each other.*

5. *The product of the divisors and the numbers in the last line will be the least common multiple.*

NOTE. — When the quotient is 1 it need not be written.

Find the least common multiple of:

2.	4, 6, and 8.	24.
3.	6, 9, and 12.	36.
4.	4, 8, and 10.	40.
5.	6, 10, and 15.	30.
6.	6, 8, 9, and 12.	72.
7.	10, 12, 15, and 20.	60.
8.	9, 15, 18, and 30.	90.
9.	12, 18, 27, and 36.	108.
10.	15, 25, 30, and 50.	150.
11.	14, 21, 30, and 35.	210.
12.	15, 20, 21, and 28.	420.
13.	20, 24, 28, and 30.	840.
14.	45, 30, 35, and 42.	630.
15.	36, 40, 45, and 50.	1800.
16.	42, 56, and 63.	504.

17. 78, 104, and 117. **936.**

18. 125, 150, and 200. **3000.**

19. 10, 24, 25, 32, and 45. **7200.**

20. 2, 3, 4, 5, 6, 7, 8, and 9. **2520.**

21. 16, 27, 42, and 108. **3024.**

22. 13, 29, 52, and 87. **4524.**

23. 120, 360, 144, 720, and 72. **720.**

CANCELLATION

91. 1. I bought 3 oranges at 5 cents each, and paid for them with pears at 3 cents each. How many pears did it take?

OPERATION

$$\begin{array}{r} 5 \\ 3 \\ 3\overline{)15} \\ 5 \end{array}$$

Solution I. — 3 times 5 cents = 15 cents, the price of the oranges. 15 cents divided by 3 cents is 5, the number of pears.

From a consideration of this example and its solution we have the following principle:

Principle. — *A number is not changed by multiplying it and then dividing the product by the multiplier.*

For the example, then, we may offer the following solution and operation:

Solution II. — Indicate the multiplication and division; then erase or *cancel* the multiplier 3 and the divisor 3 by drawing a line across them; and write the result, equal to 5.

OPERATION

$$\frac{5 \times \cancel{3}}{\cancel{3}} = 5$$

Note. — The product 5×3 forms a dividend of which 3 is the divisor.

2. If I buy 10 pears at 3 cents each, and pay for them with oranges at 5 cents each, how many oranges will it take?

SOLUTION. — 5 is a factor of 10, for $10 = 5 \times 2$. Cancel the divisor 5 and also the factor 5 in 10 by canceling 10 and writing the remaining factor 2 above it. The product of the remaining factors is 6.

OPERATION

$$\frac{\overset{2}{\cancel{10}} \times 3}{\cancel{5}} = 6$$

3. Divide 15×21 by 14×10.

SOLUTION. — 5 is a common factor of 15 and 10. Cancel 15, writing 3 above it, and 10, writing 2 below it. 7 is a common factor of 14 and 21. Cancel 14, writing 2 below it, and 21, writing 3 above it. The product of the factors remaining in the dividend is 9, and of those remaining in the divisor is 4. The quotient of 9 divided by 4 is $2\frac{1}{4}$.

OPERATION

$$\frac{\overset{3}{\cancel{15}} \times \overset{3}{\cancel{21}}}{\underset{2}{\cancel{14}} \times \underset{2}{\cancel{10}}} = \frac{9}{4} = 2\frac{1}{4}$$

Cancellation is a process of shortening computations by omitting the common factors of the dividend and divisor.

Rule. — 1. *Cancel the factors common to both dividend and divisor.*

2. *Divide the product of the factors remaining in the dividend by the product of the factors remaining in the divisor.*

3. *The result will be the quotient required.*

4. How many barrels of molasses, at $13 a barrel, will pay for 13 barrels of flour, at $4 a barrel? 4.

5. Multiply 17 by 18, and divide by 6. 51.

6. 15 times 8 = how many times 4? 30.

7. 24 times 4 = how many times 8? 12.

8. 37 times 15 = how many times 5? 111.

9. Multiply 36 by 40, and divide the product by 30 multiplied by 8. 6.

10. 36 times 5 = how many times 15? 12.

11. Divide the product of 42, 25, and 18, by the product of 21 and 15. 60.

12. I sold 23 desks, at $10 each, and was paid in $5 bills. How many did I receive? 46.

13. How many yards of flannel, at 35 cents a yard, will pay for 15 yards of gingham, at 14 cents? 6 yd.

14. What is the quotient of $21 \times 11 \times 6 \times 26$, divided by $13 \times 3 \times 14 \times 2$? 33.

15. The factors of a dividend are 21, 15, 33, 8, 14, and 17; of a divisor, 20, 34, 22, and 27. What is the quotient? 49.

16. What is the quotient of $21 \times 95 \times 6$ divided by 35×9? 38.

17. What is the quotient of $35 \times 39 \times 40$ divided by $26 \times 30 \times 42$? $1\frac{2}{3}$.

18. What is the quotient of $26 \times 33 \times 35$ divided by $4 \times 9 \times 25$? $33\frac{11}{30}$.

19. What is the quotient of $6 \times 9 \times 15 \times 21$ divided by $4 \times 6 \times 10 \times 14$? $5\frac{1}{16}$.

20. What is the quotient of $21 \times 24 \times 28 \times 35$ divided by $14 \times 18 \times 20 \times 25$? $3\frac{23}{25}$.

COMMON FRACTIONS

92. A unit may be divided into equal parts.

Thus, an apple may be divided equally between two boys, by cutting it into *two equal parts.*

An apple may be divided equally among three boys, by cutting it into *three equal parts.*

In like manner, an apple may be divided into *four, five, six, or any number of equal parts.*

These equal parts into which a unit may be divided are called *fractions.*

93. A **fraction** is one or more equal parts of a unit.

To express fractions by words and figures:

When a unit is divided into two equal parts,

Each part is called *one half,* written $\frac{1}{2}$.
Both parts are called *two halves,* written $\frac{2}{2}$.

When a unit is divided into three equal parts,

Each part is called *one third,* written $\frac{1}{3}$.
Two parts are called *two thirds,* written $\frac{2}{3}$.
All the parts are called *three thirds,* written $\frac{3}{3}$.

When a unit is divided into four equal parts,

Each part is called *one fourth,* written $\frac{1}{4}$.
Two parts are called *two fourths,* written $\frac{2}{4}$.
Three parts are called *three fourths,* written $\frac{3}{4}$.
All the parts are called *four fourths,* written $\frac{4}{4}$.

When a unit is divided into five equal parts,

Each part is called *one fifth*, written $\frac{1}{5}$.
Two parts are called *two fifths*, written $\frac{2}{5}$.
Three parts are called *three fifths*, written $\frac{3}{5}$.
Four parts are called *four fifths*, written $\frac{4}{5}$.
All the parts are called *five fifths*, written $\frac{5}{5}$.

When a unit is divided into six, seven, eight, etc., equal parts, each part is called *one sixth*, $\frac{1}{6}$, *one seventh*, $\frac{1}{7}$, *one eighth*, $\frac{1}{8}$, etc.

94. A fraction is expressed *in words* by two numbers; the first *numbers* the parts, the second *names* them; the first number is called the **numerator**, the second is called the **denominator**.

The **denominator** shows into how many equal parts the unit is divided, and the **numerator**, how many of the parts are taken.

A fraction is expressed *in figures*, by writing the numerator above the denominator with a line between them.

The numerator and denominator are styled the **terms** of the fraction.

95. When a unit is divided into equal parts, the size of each part depends upon the number of the parts.

Thus, if apples of equal size are divided, one into *two* equal parts, another into *three* equal parts, another into *four* equal parts, etc., a *half* will be larger than a *third*, a *third* larger than a *fourth*, etc. Hence,

1st. The less the number of parts into which a unit is divided, the greater the size of each part.

2d. The greater the number of parts into which a unit is divided, the less the size of each part.

96. A fraction may also be regarded as a part of one or more units.

Thus, two apples may be divided equally among three boys.

Each boy will receive, either *one third* of each of the two apples, or *two thirds* of one of the apples; therefore, $\frac{1}{3}$ of 2 is $\frac{2}{3}$. Hence, $\frac{2}{3}$ may be considered either as *two thirds* or as *one third of two*.

Two apples may be divided equally between two boys.

Each boy will receive, either *one half* of each of the two apples, or *one* of the two apples; therefore, $\frac{1}{2}$ of 2 is $\frac{2}{2}$, or 1. Hence, $\frac{2}{2}$ may be considered either as *two halves* or as *one half of two*.

Three apples may be divided equally between two boys.

Each boy will receive, either *one half* of each of the three apples, or *one apple* and *one half* of another; therefore, $\frac{1}{2}$ of 3 is $\frac{3}{2}$, or $1\frac{1}{2}$. Hence, $\frac{3}{2}$ may be considered either as *three halves* or as *one half of three*.

97. A fraction may also be regarded as an expression of division, in which the numerator is the *dividend* and the denominator the *divisor*.

Thus, $\frac{2}{3}$ is 2 divided by 3; here, the division can only be indicated.

$\frac{4}{2}$ is 4 divided by 2; in this case, the division can be performed exactly, giving a *quotient* 2.

$\frac{5}{2}$ is 5 divided by 2; in this case, the division cannot be performed exactly, the *quotient* being $2\frac{1}{2}$.

A whole number may be expressed in the form of a fraction, by writing the number for the numerator and 1 for the denominator.

Thus, 2 may be written $\frac{2}{1}$; for 2 divided by 1 is 2; 3 may be written $\frac{3}{1}$; 4 may be written $\frac{4}{1}$, etc.

98. The **value** of a fraction is its relation to a unit.

When the numerator is less than the denominator, the value of the fraction is less than 1.

Thus, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, etc., are less than 1.

When the numerator is equal to the denominator, the value of the fraction is equal to 1.

Thus, $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, etc., equal 1.

When the numerator is greater than the denominator, the value of the fraction is greater than 1.

Thus, $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$, etc., are greater than 1.

99. A **proper** fraction is one whose value is less than 1.

An **improper** fraction is one whose value is equal to or greater than 1.

A **mixed** number is a whole number and a fraction.

100. A fraction may be divided into equal parts.

Thus, after an apple has been divided into two equal parts, each half may be divided into two equal parts; the whole apple will then be divided into four equal parts; therefore, $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$.

Such expressions as $\frac{1}{2}$ of $\frac{1}{2}$, $\frac{1}{2}$ of $\frac{1}{3}$, etc., are termed **compound fractions**.

101. The operations with fractions depend upon the following principles:

Principles.—I. *A fraction is multiplied by multiplying the numerator.*

Thus, if the numerator of $\frac{2}{7}$ is multiplied by 3, the result will be $\frac{6}{7}$; in $\frac{6}{7}$ the parts are of the same size as in $\frac{2}{7}$, but there are *three times* as many.

II. *A fraction is divided by dividing the numerator.*

Thus, if the numerator of $\frac{6}{7}$ is divided by 3, the result will be $\frac{2}{7}$; in $\frac{2}{7}$ the parts are of the same size as in $\frac{6}{7}$, but there are only *one third* as many.

III. *A fraction is divided by multiplying the denominator.*

Thus, if the denominator of $\frac{2}{3}$ is multiplied by 3, the result will be $\frac{2}{9}$; in $\frac{2}{9}$ there are the same number of parts as in $\frac{2}{3}$, but the parts are only *one third* as large.

IV. *A fraction is multiplied by dividing the denominator.*

Thus, if the denominator of $\frac{2}{9}$ is divided by 3, the result will be $\frac{2}{3}$; in $\frac{2}{3}$ there are the same number of parts as in $\frac{2}{9}$, but the parts are *three times* as large.

V. *Multiplying both terms of a fraction by the same number does not change its value.*

Thus, if both terms of $\frac{3}{5}$ are multiplied by 2, the result is $\frac{6}{10}$; in $\frac{6}{10}$ there are *twice* as many parts as in $\frac{3}{5}$, but they are only *one half* as large.

VI. *Dividing both terms of a fraction by the same number does not change its value.*

Thus, if both terms of $\frac{6}{10}$ are divided by 2, the result will be $\frac{3}{5}$; in $\frac{3}{5}$ there are only *one half* as many parts as in $\frac{6}{10}$, but they are *twice* as large.

These six principles may be stated more briefly, as follows:

Principles.—I. *A fraction is* ***multiplied*** *by multiplying the numerator, or by dividing the denominator.*

II. *A fraction is* ***divided*** *by dividing the numerator, or by multiplying the denominator.*

III. *The value of a fraction is* ***not changed*** *by multiplying or dividing both terms by the same number.*

REDUCTION OF FRACTIONS

102. **Reduction of fractions** is changing their form without altering their value.

103. **To reduce an integer to an improper fraction, having a given denominator:**

1. In 3 apples, how many halves are there?

SOLUTION.—In 1 apple there are 2 halves; then, in 3 apples there are 3 × 2 halves = 6 halves.

OPERATION

$3 \times \frac{2}{2} = \frac{6}{2}$

Rule. — 1. ***Multiply the integer by the given denominator; under the product write the denominator.***

2. In 4 apples, how many halves are there? $\frac{8}{2}$.
3. In 2 apples, how many thirds are there? $\frac{6}{3}$.
4. In 3 apples, how many fourths are there? $\frac{12}{4}$.
5. In 4 apples, how many fifths are there? $\frac{20}{5}$.
6. In 6 inches, how many tenths are there? $\frac{60}{10}$.
7. In 8 feet, how many twelfths are there? $\frac{96}{12}$.
8. Reduce 4 to sevenths. $\frac{28}{7}$.
9. Reduce 8 to ninths. $\frac{72}{9}$.
10. Reduce 19 to thirteenths. $\frac{247}{13}$.
11. Reduce 25 to twentieths. $\frac{500}{20}$.
12. Reduce 37 to twenty-thirds. $\frac{851}{23}$.

104. To reduce a mixed number to an improper fraction:

1. In $3\frac{1}{2}$ apples, how many halves are there?

Solution. — In 1 apple there are 2 halves; in 3 apples there are 3×2 halves = 6 halves. 6 halves and 1 half are 7 halves.

OPERATION

$3 \times \frac{2}{2} = \frac{6}{2}$

$\frac{6}{2} + \frac{1}{2} = \frac{7}{2}$

Rule. — ***Multiply the integer by the denominator of the fraction; to the product add the numerator, and under the sum write the denominator.***

2. In $4\frac{1}{2}$ apples, how many halves are there? $\frac{9}{2}$.
3. In $2\frac{1}{3}$ apples, how many thirds are there? $\frac{7}{3}$.
4. In $2\frac{2}{3}$ apples, how many thirds are there? $\frac{8}{3}$.
5. In $5\frac{3}{4}$ dollars, how many fourths are there? $\frac{23}{4}$.
6. Reduce $8\frac{3}{4}$ to an improper fraction. $\frac{35}{4}$.
7. Reduce $12\frac{3}{5}$ to an improper fraction. $\frac{63}{5}$.

8. Reduce $15\frac{5}{6}$ to an improper fraction. $\frac{95}{6}$.

9. Reduce $26\frac{13}{24}$ to an improper fraction. $\frac{637}{24}$.

10. Reduce $3\frac{17}{55}$ to an improper fraction. $\frac{182}{55}$.

11. Reduce $46\frac{5}{8}$ to an improper fraction. $\frac{373}{8}$.

12. Reduce $21\frac{117}{583}$ to an improper fraction. $\frac{12360}{583}$.

13. Reduce $1\frac{999}{1000}$ to an improper fraction. $\frac{1999}{1000}$.

14. Reduce $14\frac{6}{71}$ to an improper fraction. $\frac{1000}{71}$.

15. Reduce $10\frac{1}{111}$ to an improper fraction. $\frac{1111}{111}$.

105. To reduce an improper fraction to an integer or a mixed number:

1. In $\frac{6}{2}$ of an apple, how many apples are there?

OPERATION

$$2)\underline{6}$$
$$3$$

Solution. — There are 2 halves in 1 apple; in 6 halves, there are $6 \div 2 = 3$ apples.

2. In $\frac{9}{4}$ of a dollar, how many dollars are there?

OPERATION

$$4)\underline{9}$$
$$2\frac{1}{4}$$

Solution. — There are 4 fourths in 1 dollar; in 9 fourths, there are $9 \div 4 = 2\frac{1}{4}$ dollars.

Rule. — *Divide the numerator by the denominator; the quotient will be the integer or the mixed number.*

3. In $\frac{6}{3}$ of an apple, how many apples are there? 2.

4. In $\frac{12}{4}$ of an apple, how many apples are there? 3.

5. In $\frac{15}{4}$ of a dollar, how many dollars are there? $\$3\frac{3}{4}$.

6. In $\frac{17}{5}$ of a dollar, how many dollars are there? $\$3\frac{2}{5}$.

7. In $\frac{19}{7}$ of a bushel, how many bushels are there? $2\frac{5}{7}$ bu.

8. In $\frac{23}{10}$ of a dollar, how many dollars are there? $\$2\frac{3}{10}$.

9. In $\frac{25}{3}$ of an ounce, how many ounces are there? $8\frac{1}{3}$ oz.

10. In $\frac{53}{4}$ of a dollar, how many dollars are there? $\$13\frac{1}{4}$.

11. Reduce $\frac{75}{4}$ to a mixed number. $18\frac{3}{4}$.

12.	Reduce $\frac{125}{8}$ to a mixed number.	$15\frac{5}{8}$.
13.	Reduce $\frac{611}{24}$ to a mixed number.	$25\frac{11}{24}$.
14.	Reduce $\frac{3000}{75}$ to an integer.	40.
15.	Reduce $\frac{775}{25}$ to an integer.	31.
16.	Reduce $\frac{171}{12}$ to a mixed number.	$14\frac{3}{12}$.
17.	Reduce $\frac{509}{11}$ to a mixed number.	$46\frac{3}{11}$.
18.	Reduce $\frac{6437}{298}$ to a mixed number.	$21\frac{179}{298}$.
19.	Reduce $\frac{7586}{125}$ to a mixed number.	$60\frac{86}{125}$.
20.	Reduce $\frac{3781}{19}$ to an integer.	199.
21.	Reduce $\frac{1325}{101}$ to a mixed number.	$13\frac{12}{101}$.

106. To reduce a fraction to higher terms:

A fraction is reduced to higher terms by multiplying both terms by the same number. This does not change its value (§ **101**, Prin. V).

1. Reduce $\frac{4}{5}$ to thirtieths.

SOLUTION. — To change any number of fifths to thirtieths we must multiply by 6, since 30 divided by 5 is 6. Multiplying both terms of $\frac{4}{5}$ by 6, the result is $\frac{24}{30}$.

OPERATION

$$30 \div 5 = 6$$
$$6 \times 4 = 24$$
$$\frac{4}{5} = \frac{24}{30}$$

Rule. — 1. *Divide the required denominator by the denominator of the given fraction.*

2. *Multiply both terms of the fraction by the quotient; the result will be the required fraction.*

2.	Reduce $\frac{1}{2}$ to fourths.	$\frac{2}{4}$.
3.	Reduce $\frac{2}{3}$ to sixths.	$\frac{4}{6}$.
4.	Reduce $\frac{3}{4}$ to twelfths.	$\frac{9}{12}$.
5.	Reduce $\frac{5}{6}$ to twenty-fourths.	$\frac{20}{24}$.
6.	Reduce $\frac{5}{7}$ to twenty-eighths.	$\frac{20}{28}$.
7.	Reduce $\frac{4}{21}$ to eighty-fourths.	$\frac{16}{84}$.

8. Reduce $\frac{7}{8}$ to seventy-seconds. $\frac{63}{72}$.

9. Reduce $\frac{3}{5}$ to sixtieths. $\frac{36}{60}$.

10. Reduce $\frac{9}{10}$ to hundredths. $\frac{90}{100}$.

11. Reduce $\frac{9}{20}$ to a fraction whose denominator is 720. $\frac{324}{720}$.

12. Reduce $\frac{13}{14}$ to a fraction whose denominator is 2016. $\frac{1872}{2016}$.

13. Reduce $\frac{22}{43}$ to a fraction whose denominator is 1935. $\frac{990}{1935}$.

14. Reduce $\frac{35}{41}$ to a fraction whose denominator is 8118. $\frac{6930}{8118}$.

15. Reduce $\frac{16}{17}$ to a fraction whose denominator is 5134. $\frac{4832}{5134}$.

16. Reduce $\frac{77}{81}$ to a fraction whose denominator is 23328. $\frac{22176}{23328}$.

17. Reduce $\frac{13}{21}$ to a fraction whose denominator is 2541. $\frac{1573}{2541}$.

107. To reduce a fraction to its lowest terms:

A fraction is reduced to lower terms by dividing both terms by the same number. This does not change its value. (§ 101, Prin. VI.)

A fraction is in its lowest terms when the numerator and denominator are prime to each other.

1. Reduce $\frac{24}{30}$ to its lowest terms.

First Method

SOLUTION. — 2 is a common factor of 24 and 30. Dividing both terms of $\frac{24}{30}$ by 2, the result is $\frac{12}{15}$. 3 is a common factor of 12 and 15. Dividing both terms of $\frac{12}{15}$ by 3, the result is $\frac{4}{5}$. 4 and 5 are prime to each other.

OPERATION

$$2)\frac{24}{30} = \frac{12}{15}$$

$$3)\frac{12}{15} = \frac{4}{5}$$

Rule.—1. ***Divide both terms of the given fraction by any common factor.***

2. ***Divide the resulting fraction in the same manner.***

3. ***So continue to divide until a fraction is obtained whose terms are prime to each other.***

Second Method

SOLUTION.—The greatest common divisor of 24 and 30 is 6. Dividing both terms of $\frac{24}{30}$ by 6, the result is $\frac{4}{5}$.

OPERATION

$$24)30(1$$
$$\underline{24}$$
$$6)24(4$$
$$\underline{24}$$
$$6)\frac{24}{30} = \frac{4}{5}$$

Rule.—1. ***Divide both terms of the given fraction by their greatest common divisor.***

2. ***The resulting fraction will be in its lowest terms.***

Reduce to lowest terms:

2.	$\frac{18}{30}$	$\frac{3}{5}$.	**11.**	$\frac{182}{196}$	$\frac{13}{14}$.
3.	$\frac{60}{90}$	$\frac{2}{3}$.	**12.**	$\frac{615}{915}$	$\frac{41}{61}$.
4.	$\frac{12}{18}$	$\frac{2}{3}$.	**13.**	$\frac{873}{1067}$	$\frac{9}{11}$.
5.	$\frac{30}{45}$	$\frac{2}{3}$.	**14.**	$\frac{777}{1998}$	$\frac{7}{18}$.
6.	$\frac{60}{150}$	$\frac{2}{5}$.	**15.**	$\frac{909}{2323}$	$\frac{9}{23}$.
7.	$\frac{42}{70}$	$\frac{3}{5}$.	**16.**	$\frac{391}{667}$	$\frac{17}{29}$.
8.	$\frac{96}{112}$	$\frac{6}{7}$.	**17.**	$\frac{585}{1287}$	$\frac{5}{11}$.
9.	$\frac{60}{125}$	$\frac{12}{25}$.	**18.**	$\frac{796}{14129}$	$\frac{4}{71}$.
10.	$\frac{126}{198}$	$\frac{7}{11}$.	**19.**	$\frac{1457}{5921}$	$\frac{47}{191}$.

108. **To reduce two or more fractions to their least common denominator:**

Two or more fractions have a *common denominator* when they have the same denominator.

A **common denominator** of two or more fractions is a common multiple of their denominators.

The **least common denominator** of two or more fractions is the least common multiple of their denominators.

1. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$, and $\frac{11}{12}$ to their least common denominator.

SOLUTION. — The least common multiple of the denominators 4, 6, 9, and 12, is the L. C. M. of 9 and 12, which is 36 (§ **90**). Each fraction, then, must be reduced to thirty-sixths (§ **106**). $\frac{3}{4} = \frac{27}{36}$, $\frac{5}{6} = \frac{30}{36}$, $\frac{8}{9} = \frac{32}{36}$, and $\frac{11}{12} = \frac{33}{36}$.

OPERATION

$$3 \,\underline{|\ 9 \quad 12}$$
$$\quad 3 \quad 4$$

$3 \times 3 \times 4 = 36$, L. C. M.

$36 \div 4 = 9$	$36 \div 6 = 6$
$9 \times 3 = 27$	$6 \times 5 = 30$
$\frac{3}{4} = \frac{27}{36}$	$\frac{5}{6} = \frac{30}{36}$
$36 \div 9 = 4$	$36 \div 12 = 3$
$4 \times 8 = 32$	$3 \times 11 = 33$
$\frac{8}{9} = \frac{32}{36}$	$\frac{11}{12} = \frac{33}{36}$

Rule. — 1. *Find the L. C. M. of the denominators of the fractions for their least common denominator.*

2. *Reduce each fraction to another having this denominator.*

NOTES. — 1. Integers must be reduced to the common denominator (§ **103**, Rule).

2. Before commencing the operation mixed numbers must be reduced to improper fractions (§ **104**).

3. Each fraction must be in its lowest terms (§ **107**).

4. Two or more fractions may be reduced to any common denominator in the same way.

Reduce to their least common denominator:

2.	$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.	$\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$.
3.	$\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{9}$.	$\frac{12}{18}$, $\frac{15}{18}$, $\frac{14}{18}$.
4.	$\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$.	$\frac{10}{20}$, $\frac{15}{20}$, $\frac{16}{20}$.
5.	$\frac{3}{8}$, $\frac{4}{5}$, $\frac{9}{10}$.	$\frac{15}{40}$, $\frac{32}{40}$, $\frac{36}{40}$.

6. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$. $\frac{16}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

7. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{5}{9}$. $\frac{54}{72}$, $\frac{45}{72}$, $\frac{40}{72}$.

8. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$. $\frac{4}{8}$, $\frac{6}{8}$, $\frac{7}{8}$.

9. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{12}$. $\frac{8}{12}$, $\frac{10}{12}$, $\frac{7}{12}$.

10. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{11}{16}$. $\frac{12}{16}$, $\frac{10}{16}$, $\frac{11}{16}$.

11. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

12. $\frac{2}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{5}{8}$. $\frac{560}{840}$, $\frac{336}{840}$, $\frac{360}{840}$, $\frac{525}{840}$.

13. $\frac{2}{7}$, $\frac{5}{14}$, $\frac{9}{21}$, $\frac{11}{28}$. $\frac{8}{28}$, $\frac{10}{28}$, $\frac{12}{28}$, $\frac{11}{28}$.

14. $\frac{2}{5}$, $\frac{3}{4}$, $\frac{6}{9}$, $\frac{15}{18}$. $\frac{24}{60}$, $\frac{45}{60}$, $\frac{40}{60}$, $\frac{50}{60}$.

15. 2, $\frac{3}{4}$, $\frac{5}{9}$, $\frac{7}{12}$. $\frac{72}{36}$, $\frac{27}{36}$, $\frac{20}{36}$, $\frac{21}{36}$.

16. $2\frac{2}{3}$, $\frac{3}{5}$, 4, $5\frac{5}{6}$. $\frac{80}{30}$, $\frac{18}{30}$, $\frac{120}{30}$, $\frac{175}{30}$.

17. $2\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{4}$, 5. $\frac{30}{12}$, $\frac{40}{12}$, $\frac{51}{12}$, $\frac{60}{12}$.

18. $\frac{7}{16}$, $\frac{11}{18}$, $\frac{17}{24}$, $\frac{19}{36}$, $\frac{25}{48}$. $\frac{63}{144}$, $\frac{88}{144}$, $\frac{102}{144}$, $\frac{76}{144}$, $\frac{75}{144}$.

19. $\frac{4}{7}$, $\frac{3}{10}$, $\frac{5}{12}$, $\frac{17}{35}$, $\frac{4}{63}$, $\frac{15}{28}$.

$\frac{720}{1260}$, $\frac{378}{1260}$, $\frac{525}{1260}$, $\frac{612}{1260}$, $\frac{80}{1260}$, $\frac{675}{1260}$.

20. $\frac{3}{5}$, $\frac{7}{10}$, $\frac{6}{25}$, $\frac{11}{30}$, $\frac{13}{45}$, $\frac{23}{60}$.

$\frac{540}{900}$, $\frac{630}{900}$, $\frac{216}{900}$, $\frac{330}{900}$, $\frac{260}{900}$, $\frac{345}{900}$.

ADDITION OF FRACTIONS

109. Addition of fractions is the process of finding the sum of two or more fractional numbers. There are *two cases.*

110. When the fractions have a common denominator:

1. Add $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{3}{5}$.

OPERATION

$$\frac{1}{5} + \frac{2}{5} + \frac{3}{5} = \frac{6}{5}$$
$$\frac{6}{5} = 1\frac{1}{5}$$

SOLUTION. — The sum of 1 fifth, 2 fifths, and 3 fifths, is 6 fifths. $\frac{6}{5}$ are equal to $1\frac{1}{5}$ (§ **105**).

EXPLANATION.—Since the denominators are the same, the numerators express parts of the same size; therefore, add 1 fifth, 2 fifths, and 3 fifths, as you would add 1 cent, 2 cents, and 3 cents; the sum, in one case, being 6 fifths, in the other, 6 cents.

Rule.—*Add the numerators; under the sum write the common denominator.*

NOTES.—1. The result, if an improper fraction, must be reduced to an integer, or a mixed number (§ **105**).

2. The result must be reduced to its lowest terms (§ **107**).

2.	Add $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$.	$1\frac{1}{2}$.
3.	Add $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.	2.
4.	Add $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{6}{7}$.	$1\frac{5}{7}$.
5.	Add $\frac{4}{9}$, $\frac{5}{9}$, $\frac{7}{9}$, $\frac{8}{9}$.	$2\frac{2}{3}$.
6.	Add $\frac{3}{11}$, $\frac{7}{11}$, $\frac{8}{11}$, $\frac{10}{11}$.	$2\frac{6}{11}$.
7.	Add $\frac{5}{13}$, $\frac{8}{13}$, $\frac{9}{13}$, $\frac{11}{13}$.	$2\frac{7}{13}$.
8.	Add $\frac{7}{15}$, $\frac{8}{15}$, $\frac{11}{15}$, $\frac{13}{15}$.	$2\frac{3}{5}$.
9.	Add $\frac{9}{20}$, $\frac{11}{20}$, $\frac{13}{20}$, $\frac{17}{20}$.	$2\frac{1}{2}$.
10.	Add $\frac{12}{25}$, $\frac{16}{25}$, $\frac{18}{25}$, $\frac{24}{25}$.	$2\frac{4}{5}$.

111. When the fractions have not a common denominator:

1. Add $\frac{5}{6}$, $\frac{8}{9}$, and $\frac{11}{12}$.

SOLUTION.—Reducing the fractions to a common denominator (§ **108**), $\frac{5}{6}=\frac{30}{36}$, $\frac{8}{9}=\frac{32}{36}$, and $\frac{11}{12}=\frac{33}{36}$; the sum of $\frac{30}{36}$, $\frac{32}{36}$, and $\frac{33}{36}$ is $\frac{95}{36}$. $\frac{95}{36}$ are equal to $2\frac{23}{36}$.

OPERATION

$$\frac{5}{6}=\frac{30}{36}\quad \frac{8}{9}=\frac{32}{36}\quad \frac{11}{12}=\frac{33}{36}$$

$$\frac{30}{36}+\frac{32}{36}+\frac{33}{36}=\frac{95}{36}$$

$$\frac{95}{36}=2\frac{23}{36}$$

EXPLANATION.—Since the denominators are different, the numerators do not express parts of the same size; therefore, the fractions cannot be added until they are reduced to a common denominator.

Rule.—1. *Reduce the fractions to a common denominator.*

2. *Add the numerators, and under the sum write the common denominator.*

NOTES.—1. Integers and fractions may be added separately and their sums then united.

2. The integral and the fractional parts of mixed numbers may be added separately and their sums then united.

2. Add $\frac{1}{2}$ and $\frac{1}{3}$. $\frac{5}{6}$.

3. Add $\frac{1}{3}$ and $\frac{1}{4}$. $\frac{7}{12}$.

4. Add $\frac{1}{2}$ and $\frac{3}{5}$. $1\frac{1}{10}$.

5. Add $\frac{5}{6}$ and $\frac{1}{9}$. $\frac{17}{18}$.

6. Add $\frac{3}{4}$ and $\frac{5}{6}$. $1\frac{7}{12}$.

7. Add $\frac{7}{8}$ and $\frac{11}{12}$. $1\frac{19}{24}$.

8. Add $2\frac{1}{2}$ and $3\frac{2}{3}$.

SOLUTION.—The sum of $\frac{1}{2}$ and $\frac{2}{3}$ is $\frac{7}{6}$; $\frac{7}{6} = 1\frac{1}{6}$; write the $\frac{1}{6}$ under the column of fractions and add the 1 to the column of integers. The sum of 1, 3, and 2 is 6.

OPERATION

$$\begin{array}{l} 2\frac{1}{2} \\ \underline{3\frac{2}{3}} \\ 6\frac{1}{6} \textit{ Ans.} \end{array}$$

9. Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$. $2\frac{1}{4}$.

10. Add $\frac{1}{4}$, $\frac{7}{8}$, $\frac{11}{12}$. $2\frac{1}{24}$.

11. Add $\frac{1}{8}$, $\frac{1}{9}$, $\frac{2}{11}$. $\frac{331}{792}$.

12. Add $\frac{4}{5}$, $7\frac{1}{2}$, $8\frac{3}{4}$. $17\frac{1}{20}$.

13. Add $\frac{1}{12}$, $\frac{1}{13}$, $\frac{1}{14}$, $\frac{1}{15}$. $\frac{543}{1820}$.

14. Add $\frac{13}{18}$, $\frac{8}{15}$, $\frac{11}{20}$, $\frac{13}{30}$. $2\frac{43}{180}$.

15. Add $\frac{7}{12}$, $2\frac{5}{6}$, $3\frac{3}{8}$, $3\frac{4}{9}$. $10\frac{17}{72}$.

16. Add $16\frac{2}{3}$, $12\frac{3}{4}$, $8\frac{3}{5}$, $2\frac{1}{4}$. $40\frac{4}{15}$.

17. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$. $1\frac{9}{20}$.

18. Add $\frac{2}{5}$, $\frac{7}{16}$, $\frac{7}{50}$, $\frac{3}{140}$, $\frac{3}{2800}$. 1.

19. Add $\frac{1}{20}$, $\frac{7}{16}$, $\frac{11}{12}$, $1\frac{2}{15}$, $2\frac{11}{18}$. $5\frac{107}{720}$.

20. Add $\frac{2}{3}$, $2\frac{1}{2}$, $4\frac{1}{5}$, $6\frac{1}{3}$, $8\frac{1}{4}$. $21\frac{19}{20}$.

21. Add $1\frac{1}{3}$, $4\frac{2}{7}$, $2\frac{1}{5}$, $2\frac{1}{21}$. $9\frac{13}{15}$.

SUBTRACTION OF FRACTIONS

112. Subtraction of fractions is the process of finding the difference between two fractional numbers. There are *two cases.*

113. When the fractions have a common denominator:

1. From $\frac{5}{7}$ subtract $\frac{2}{7}$.

OPERATION

$$\tfrac{5}{7} - \tfrac{2}{7} = \tfrac{3}{7}$$

SOLUTION. — 2 sevenths from 5 sevenths leaves 3 sevenths.

EXPLANATION. — Since the denominators are the same, the numerators express parts of the same size; therefore, subtract 2 sevenths from 5 sevenths as you would subtract 2 cents from 5 cents; the remainder, in one case, being 3 sevenths, in the other 3 cents.

Rule. — *From the greater numerator subtract the less; under the remainder write the common denominator.*

2. From $\frac{3}{4}$ subtract $\frac{1}{4}$. $\frac{1}{2}$.
3. From $\frac{7}{8}$ subtract $\frac{5}{8}$. $\frac{1}{4}$.
4. From $\frac{5}{9}$ subtract $\frac{2}{9}$. $\frac{1}{3}$.
5. From $\frac{8}{10}$ subtract $\frac{3}{10}$. $\frac{1}{2}$.
6. From $3\frac{1}{8}$ subtract $1\frac{3}{8}$.

OPERATION

$$\begin{array}{r} 3\frac{1}{8} \\ 1\frac{3}{8} \\ \hline 1\frac{3}{4} \end{array} \textit{ Ans.}$$

SOLUTION. — $\frac{3}{8}$ cannot be taken from $\frac{1}{8}$; so change 1 of the 3 integers to eighths. 1 equals $\frac{8}{8}$; $\frac{8}{8}$ and $\frac{1}{8}$ are $\frac{9}{8}$; $\frac{3}{8}$ from $\frac{9}{8}$ leaves $\frac{6}{8}$; $\frac{6}{8} = \frac{3}{4}$. 1 from 2 leaves 1.

7. From $4\frac{1}{4}$ subtract $2\frac{3}{4}$. $1\frac{1}{2}$.
8. From $8\frac{1}{3}$ subtract $3\frac{2}{3}$. $4\frac{2}{3}$.
9. From $23\frac{7}{20}$ subtract $17\frac{11}{20}$. $5\frac{4}{5}$.

114. When the fractions have not a common denominator:

1. From $\frac{9}{10}$ subtract $\frac{5}{6}$.

SOLUTION. — Reducing the fractions to a common denominator (§ **108**), $\frac{5}{6}=\frac{25}{30}$ and $\frac{9}{10}=\frac{27}{30}$; then, $\frac{25}{30}$ from $\frac{27}{30}$ leaves $\frac{2}{30}$; $\frac{2}{30}=\frac{1}{15}$.

OPERATION

$$\frac{9}{10}=\frac{27}{30} \quad \frac{5}{6}=\frac{25}{30}$$
$$\frac{27}{30}-\frac{25}{30}=\frac{2}{30}$$
$$\frac{2}{30}=\frac{1}{15}$$

EXPLANATION. — Since the denominators are different, the numerators do not express parts of the same size; therefore, one fraction cannot be subtracted from the other until they are reduced to a common denominator.

Rule. — 1. *Reduce the fractions to a common denominator.*
2. *From the greater numerator subtract the less, and under the remainder write the common denominator.*

2. From $\frac{1}{2}$ subtract $\frac{1}{3}$. $\frac{1}{6}$.
3. From $\frac{1}{3}$ subtract $\frac{1}{4}$. $\frac{1}{12}$.
4. From $\frac{3}{4}$ subtract $\frac{2}{3}$. $\frac{1}{12}$.
5. From $\frac{4}{5}$ subtract $\frac{1}{2}$. $\frac{3}{10}$.
6. From $\frac{5}{6}$ subtract $\frac{3}{10}$. $\frac{8}{15}$.
7. From $\frac{5}{6}$ subtract $\frac{3}{8}$. $\frac{11}{24}$.
8. From $\frac{5}{9}$ subtract $\frac{1}{6}$. $\frac{7}{18}$.
9. From $\frac{4}{15}$ subtract $\frac{1}{10}$. $\frac{1}{6}$.
10. From $\frac{16}{21}$ subtract $\frac{5}{14}$. $\frac{17}{42}$.
11. From $3\frac{1}{2}$ subtract $1\frac{2}{3}$.

SOLUTION. — $\frac{1}{2}$ equals $\frac{3}{6}$, and $\frac{2}{3}$ equals $\frac{4}{6}$. $\frac{4}{6}$ cannot be taken from $\frac{3}{6}$; so change 1 of the three integers to sixths. 1 equals $\frac{6}{6}$; $\frac{6}{6}$ and $\frac{3}{6}$ are $\frac{9}{6}$; $\frac{4}{6}$ from $\frac{9}{6}$ leaves $\frac{5}{6}$. 1 from 2 leaves 1.

OPERATION

$$\begin{array}{r} 3\frac{1}{2} \\ 1\frac{2}{3} \\ \hline 1\frac{5}{6} \end{array} \textit{ Ans.}$$

12. From 5 subtract $\frac{2}{3}$. $4\frac{1}{3}$.
13. From $5\frac{2}{3}$ subtract $4\frac{1}{2}$. $1\frac{1}{6}$.
14. From $7\frac{2}{3}$ subtract $4\frac{3}{4}$. $2\frac{11}{12}$.

15. From $14\frac{1}{4}$ subtract $12\frac{2}{3}$. $1\frac{7}{12}$.

16. From $5\frac{3}{14}$ subtract $2\frac{10}{21}$. $2\frac{31}{42}$.

17. From $4\frac{1}{24}$ subtract $3\frac{1}{16}$. $\frac{47}{48}$.

18. From $56\frac{1}{3}$ subtract $42\frac{1}{4}$. $14\frac{1}{12}$.

19. From $60\frac{4}{5}$ subtract $41\frac{3}{10}$. $19\frac{1}{2}$.

20. From $97\frac{1}{2}$ subtract $48\frac{5}{6}$. $48\frac{2}{3}$.

MULTIPLICATION OF FRACTIONS

115. Multiplication of fractions is the process of finding the product of two or more fractional numbers.

1. If 1 apple costs $\frac{4}{5}$ of a cent, how much will 3 apples cost?

Solution. — They will cost 3 times $\frac{4}{5}$ of a cent $= \frac{12}{5}$ of a cent (§ **101**, Prin. I). $\frac{12}{5}$ equals $2\frac{2}{5}$.

OPERATION

$\frac{3}{1} \times \frac{4}{5} = \frac{12}{5}$

$\frac{12}{5} = 2\frac{2}{5}$

Explanation. — 3 apples will cost $\frac{4}{5} + \frac{4}{5} + \frac{4}{5} = \frac{12}{5}$ of a cent; hence, 3 times $\frac{4}{5} = \frac{12}{5}$.

2. What is $\frac{2}{5}$ of 12?

Solution. — $\frac{1}{5}$ of $12 = \frac{12}{5}$; $\frac{2}{5}$ of $12 = 2$ times $\frac{12}{5} = \frac{24}{5}$ (Ex. 1). $\frac{24}{5} = 4\frac{4}{5}$.

OPERATION

$\frac{2}{5} \times \frac{12}{1} = \frac{24}{5}$

$\frac{24}{5} = 4\frac{4}{5}$

3. What is $\frac{4}{7}$ of $\frac{3}{5}$?

Solution. — $\frac{1}{7}$ of $\frac{3}{5} = \frac{3}{35}$; $\frac{4}{7}$ of $\frac{3}{5} = 4$ times $\frac{3}{35} = \frac{12}{35}$.

OPERATION

$\frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$

Explanation. — $\frac{1}{7}$ of $\frac{1}{5}$ is $\frac{1}{35}$ (§ **100**), $\frac{1}{7}$ of $\frac{3}{5}$ is 3 times $\frac{1}{35} = \frac{3}{35}$, and $\frac{4}{7}$ of $\frac{3}{5}$ is 4 times $\frac{3}{35} = \frac{12}{35}$.

4. Multiply $\frac{2}{3}$ by $\frac{4}{5}$.

Solution. — $\frac{4}{5}$ is the same as $\frac{1}{5}$ of 4 (§ **96**). $\frac{2}{3}$ multiplied by 4 is $\frac{8}{3}$ (§ **101**, Prin. I); then, $\frac{2}{3}$ multiplied by $\frac{1}{5}$ of 4 is $\frac{1}{5}$ of $\frac{8}{3} = \frac{8}{15}$.

OPERATION

$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$

Rule. — 1. *Multiply together the numerators of the given fractions for the numerator of the product.*

2. *Multiply together the denominators of the given fractions for the denominator of the product.*

NOTES. — 1. Express integers in the form of fractions (§ **97**).

2. Reduce mixed numbers to improper fractions (§ **104**). Sometimes it may be more convenient to multiply by the integral and fractional parts separately.

3. Indicate the operation and apply the Rule for Cancellation wherever it is practicable (§ **91**).

Multiply:

5. $\frac{3}{4}$ by 3.	$2\frac{1}{4}$.	**8.** $\frac{2}{3}$ by 4.	$2\frac{2}{3}$.
6. 8 by $\frac{2}{3}$.	$5\frac{1}{3}$.	**9.** 5 by $\frac{3}{4}$.	$3\frac{3}{4}$
7. $\frac{3}{4}$ by $\frac{5}{7}$.	$\frac{15}{28}$.	**10.** $\frac{8}{9}$ by $\frac{3}{4}$.	

OPERATION

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$$

SOLUTION. — Indicating the operation and applying the Rule for Cancellation (§ **91**), the result is $\frac{2}{3}$.

Multiply:

11. $\frac{2}{3}$ by 6.	4.	**16.** $\frac{9}{13}$ by $\frac{3}{7}$.	$\frac{27}{91}$.
12. 20 by $\frac{3}{4}$.	15.	**17.** $\frac{3}{7}$ by 6.	$2\frac{4}{7}$.
13. $\frac{8}{13}$ by $\frac{11}{16}$.	$\frac{11}{26}$.	**18.** 7 by $\frac{2}{3}$.	$4\frac{2}{3}$.
14. $\frac{3}{5}$ by 10.	6.	**19.** $2\frac{1}{4}$ by $3\frac{1}{2}$.	
15. 12 by $\frac{2}{3}$.	8.		

OPERATION

$$2\frac{1}{4} = \frac{9}{4} \quad 3\frac{1}{2} = \frac{7}{2}$$
$$\frac{7}{2} \times \frac{9}{4} = \frac{63}{8}$$
$$\frac{63}{8} = 7\frac{7}{8}$$

SOLUTION. — Reducing $2\frac{1}{4}$ and $3\frac{1}{2}$ to improper fractions (§ **104**), they are $\frac{9}{4}$ and $\frac{7}{2}$. Multiplying $\frac{9}{4}$ by $\frac{7}{2}$, the result is $\frac{63}{8} = 7\frac{7}{8}$.

20. Multiply $18\frac{3}{4}$ by 8.

OPERATION

$$\begin{array}{r} 18\frac{3}{4} \\ \underline{8} \\ 144 \\ \underline{6} \\ 150 \end{array}$$

SOLUTION. — 8 times 18 is 144. 8 times $\frac{3}{4}$ is 6. $144 + 6 = 150$.

Multiply:

21. 8 by $3\frac{2}{3}$. $29\frac{1}{3}$.

22. $2\frac{1}{2}$ by $2\frac{1}{2}$. $6\frac{1}{4}$.

23. $10\frac{7}{9}$ by 7. $75\frac{4}{9}$.

24. 25 by $8\frac{3}{5}$. 215.

25. $\frac{9}{10}$ by $17\frac{3}{11}$. $15\frac{6}{11}$.

26. $10\frac{5}{6}$ by 9. $97\frac{1}{2}$.

27. 64 by $8\frac{7}{8}$. 568.

28. $8\frac{3}{4}$ by $\frac{3}{7}$. $3\frac{3}{4}$.

Find the product of:

29. $\frac{5}{12}$, $\frac{9}{16}$, $2\frac{2}{11}$. $\frac{45}{88}$.

30. $2\frac{1}{16}$, $\frac{8}{11}$, $1\frac{7}{9}$. 1.

31. $6\frac{3}{4}$, $2\frac{8}{9}$, 21. $409\frac{1}{2}$.

32. $2\frac{1}{2}$, $3\frac{2}{3}$, $4\frac{3}{4}$, $1\frac{1}{7}$. $49\frac{16}{21}$.

33. $2\frac{1}{5}$, $2\frac{3}{26}$, $3\frac{1}{4}$, $1\frac{5}{11}$. 22.

34. $\frac{7}{8}$, $\frac{8}{10}$, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{2}{8}$, $\frac{6}{7}$. $\frac{1}{9}$.

35. $\frac{1}{4}$, $\frac{9}{7}$, $\frac{4}{5}$, $\frac{7}{9}$, $\frac{5}{4}$, $\frac{2}{3}$, 6. 1.

36. $\frac{6}{7}$, $\frac{4}{9}$, $1\frac{3}{4}$, $\frac{1}{6}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{5}$, 20. $\frac{5}{9}$.

37. $2\frac{1}{2}$, $6\frac{2}{5}$, $3\frac{1}{4}$, $\frac{7}{18}$, 2, $\frac{8}{7}$. 24.

116. Fractional parts of integers are obtained by multiplication.

1. What is $\frac{2}{3}$ of 2?

OPERATION

$\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$

$\frac{4}{3} = 1\frac{1}{3}$

SOLUTION.—$\frac{1}{3}$ of 2 is $\frac{2}{3}$ (§ 96); $\frac{2}{3}$ of $2 = 2$ times $\frac{2}{3} = \frac{4}{3}$. $\frac{4}{3} = 1\frac{1}{3}$.

2. $\frac{3}{4}$ of $5 = ?$ $3\frac{3}{4}$.
3. $\frac{2}{5}$ of $7 = ?$ $2\frac{4}{5}$.
4. $\frac{4}{5}$ of $10 = ?$ 8.
5. $\frac{5}{6}$ of $12 = ?$ 10.
6. $\frac{5}{6}$ of $15 = ?$ $12\frac{1}{2}$.
7. $\frac{8}{9}$ of $21 = ?$ $18\frac{2}{3}$.
8. $\frac{7}{10}$ of $25 = ?$ $17\frac{1}{2}$.
9. $\frac{5}{12}$ of $27 = ?$ $11\frac{1}{4}$.

117. 1. Find the value of $\frac{2}{3}$ of $\frac{4}{5}$.

SOLUTION. — Multiplying $\frac{4}{5}$ by $\frac{2}{3}$ (§ 115, Rule), the result is $\frac{8}{15}$.

OPERATION
$\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.

Find the value of:

2.	$\frac{2}{3}$ of $\frac{5}{7}$.	$\frac{10}{21}$.	10.	$\frac{3}{5}$ of $\frac{5}{7}$ of $\frac{7}{8}$.	$\frac{3}{8}$.
3.	$\frac{3}{5}$ of $\frac{7}{8}$.	$\frac{21}{40}$.	11.	$\frac{3}{5}$ of $\frac{4}{9}$ of $\frac{7}{12}$ of $\frac{18}{35}$.	$\frac{2}{25}$.
4.	$\frac{1}{2}$ of $\frac{3}{5}$ of $2\frac{3}{4}$.	$\frac{33}{40}$.	12.	$\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{4}{9}$.	$\frac{1}{9}$.
5.	$\frac{7}{11}$ of $\frac{2}{3}$.	$\frac{14}{33}$.	13.	$\frac{1}{9}$ of $\frac{3}{4}$ of $1\frac{1}{3}$.	$\frac{1}{9}$.
6.	$\frac{3}{4}$ of $\frac{5}{8}$.	$\frac{15}{32}$.	14.	$\frac{3}{5}$ of $\frac{6}{7}$ of $1\frac{17}{18}$.	1.
7.	$\frac{2}{3}$ of $\frac{5}{7}$ of $1\frac{4}{9}$.	$\frac{130}{189}$.	15.	$\frac{3}{7}$ of $2\frac{2}{3}$ of $1\frac{3}{4}$.	2.
8.	$\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.	$\frac{2}{5}$.	16.	$\frac{9}{13}$ of $\frac{7}{18}$ of $1\frac{6}{7}$.	$\frac{1}{2}$.
9.	$\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$.	$\frac{5}{24}$.	17.	$\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{1}{8}$ of 5.	$\frac{1}{4}$.

18. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{5}{9}$ of $\frac{9}{10}$ to a simple fraction. $\frac{1}{16}$.

Miscellaneous Examples

118. *Find the cost of:*

1. $2\frac{1}{4}$ lb. of fish at $13\frac{1}{2}$¢ a pound. $30\frac{3}{8}$¢.
2. $6\frac{1}{2}$ yd. of silk at \$$\frac{3}{4}$ a yard. \$$4\frac{7}{8}$.
3. $3\frac{1}{2}$ lb. of sugar at $4\frac{3}{4}$¢ a pound. $16\frac{5}{8}$¢.
4. At the rate of $5\frac{1}{2}$ miles an hour, how far will a man travel in $7\frac{3}{4}$ hours? $42\frac{5}{8}$ mi.
5. I own $\frac{2}{3}$ of a steamboat, and sell $\frac{3}{5}$ of my share. What part of the boat do I sell? $\frac{2}{5}$.
6. At \$$6\frac{3}{4}$ per yard, find the cost of $\frac{1}{3}$ of a remnant of velvet containing $5\frac{1}{4}$ yd. \$$11\frac{13}{16}$.
7. $\frac{3}{7}$ of $\frac{5}{9}$ of $16\frac{1}{2} \times \frac{2}{3}$ of $\frac{7}{8}$ of $15 =$ what? $34\frac{3}{8}$.
8. What is the sum of $\frac{2}{3} + \frac{3}{4}$ and $\frac{2}{3} \times \frac{3}{4}$? $1\frac{11}{12}$.

DIVISION OF FRACTIONS

119. Division of Fractions is the process of finding the quotient of two fractional numbers.

1. If 3 equal lines measure $\frac{6}{7}$ of an inch, how much will 1 line measure?

OPERATION

$$\frac{1}{\cancel{3}} \times \frac{\overset{2}{\cancel{6}}}{7} = \frac{2}{7}$$

SOLUTION. — 1 line will measure $\frac{1}{3}$ of $\frac{6}{7} = \frac{2}{7}$ of an inch (§ **117**).

EXPLANATION. — $\frac{6}{7}$ is to be divided into 3 equal parts. Each part will be $\frac{2}{7}$ (§ **101**, Prin. II); for $\frac{6}{7} = \frac{2}{7} + \frac{2}{7} + \frac{2}{7}$.

2. At 2 dollars a yard, what part of a yard of cloth can be bought for $\frac{3}{5}$ of a dollar?

SOLUTION. — For 1 dollar $\frac{1}{2}$ a yard can be bought, and for $\frac{1}{5}$ of a dollar $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$ of a yard (§ **117**); then, for $\frac{3}{5}$ of a dollar 3 times $\frac{1}{10} = \frac{3}{10}$ of a yard can be bought.

OPERATION

$$\frac{3}{5} \times \frac{1}{2} = \frac{3}{10}$$

EXPLANATION. — Were it required to find how many yards, at \$2 a yard, could be bought for \$6, then 6 would be divided by 2; hence, to find the part of a yard that \$$\frac{3}{5}$ will pay for, $\frac{3}{5}$ must be divided by 2. To divide $\frac{3}{5}$ by 2, multiply the denominator (§ **101**, Prin. III).

3. At $\frac{2}{3}$ of a dollar for 1 yard of silk, how many yards can be bought for 4 dollars?

SOLUTION. — For $\frac{1}{3}$ of a dollar $\frac{1}{2}$ a yard can be bought, and for $\frac{3}{3}$, or 1 dollar, 3 times $\frac{1}{2} = \frac{3}{2}$ of a yard; then, for 4 dollars, there can be bought 4 times $\frac{3}{2} = 6$ yards.

OPERATION

$$\frac{\overset{2}{\cancel{4}}}{1} \times \frac{3}{\cancel{2}} = 6$$

4. At $\frac{2}{3}$ of a dollar for 1 yard of silk, how many yards can be bought for $\frac{4}{5}$ of a dollar?

SOLUTION. — For $\frac{1}{3}$ of a dollar $\frac{1}{2}$ a yard can be bought, and for $\frac{3}{3}$, or 1 dollar, 3 times $\frac{1}{2} = \frac{3}{2}$ of a yard; then, for $\frac{4}{5}$ of a dollar $\frac{4}{5}$ of $\frac{3}{2} = \frac{6}{5} = 1\frac{1}{5}$ yards can be bought (§ **117**).

OPERATION

$$\frac{4}{5} \times \frac{3}{2} = \frac{6}{5}$$

$$\frac{6}{5} = 1\frac{1}{5}$$

5. Divide $\frac{3}{4}$ by $\frac{4}{5}$.

SOLUTION. — $\frac{4}{5}$ is the same as $\frac{1}{5}$ of 4 (§ 96). $\frac{3}{4}$ divided by 4 is $\frac{3}{16}$ (§ 101, Prin. III); then, $\frac{3}{4}$ divided by $\frac{1}{5}$ of 4 is 5 times $\frac{3}{16} = \frac{15}{16}$ (§ 115).

OPERATION

$\frac{3}{4} \times \frac{5}{4} = \frac{15}{16}$

Rule. — *Multiply the dividend by the divisor with its terms inverted.*

NOTES. — 1. Express integers in the form of fractions.

2. Reduce mixed numbers to improper fractions.

3. Indicate the operation and apply the Rule for Cancellation whenever it is practicable (§ 91, Rule).

6. If 4 yards of ribbon cost $\frac{4}{5}$ of a dollar, how much will 1 yard cost? $\$\frac{1}{5}$.

7. At $\frac{1}{2}$ a cent each, how many apples can be bought for 3 cents? 6.

8. At $\frac{1}{5}$ of a dollar per yard, how many yards of lawn can be bought for $\$\frac{9}{10}$? $4\frac{1}{2}$.

9. If you can ride a certain distance for \$3, what part of the distance can you ride for $\$\frac{1}{2}$? $\frac{1}{6}$.

10. At $\frac{3}{4}$ of a dollar per yard, how many yards of cloth can you buy for 6 dollars? 8 yd.

11. At $\frac{1}{5}$ of a dollar per yard, how many yards of ribbon can be purchased for $\frac{3}{4}$ of a dollar? $3\frac{3}{4}$ yd.

12. If 7 pounds of rice cost $\frac{14}{25}$ of a dollar, how much will 1 pound cost? $\$\frac{2}{25}$.

13. At $\frac{1}{2}$ a dollar per yard, how many yards of silk can be bought for $\$3\frac{1}{4}$? $6\frac{1}{2}$ yd.

14. At $\frac{3}{5}$ of a dollar per pound, how many pounds of tea can be purchased for $\$2\frac{3}{10}$? $3\frac{5}{6}$ lb.

15. At $3\frac{3}{4}$ dollars per yard for cloth, how many yards can be purchased with \$$42\frac{1}{2}$? $11\frac{1}{3}$ yd.

16. By what must $\frac{3}{8}$ be multiplied that the product may be 10? $26\frac{2}{3}$.

17. Divide $3\frac{3}{7}$ by $\frac{3}{7}$ of $1\frac{1}{2}$. $5\frac{1}{3}$.

18. Divide $\frac{4}{11}$ of $27\frac{1}{2}$ by $\frac{3}{10}$ of $21\frac{1}{4}$. $1\frac{29}{51}$.

19. Divide $4\frac{1}{2}$ by $1\frac{1}{3}$.

OPERATION

$4\frac{1}{2} = \frac{9}{2}$
$1\frac{1}{3} = \frac{4}{3}$
$\frac{9}{2} \times \frac{3}{4} = \frac{27}{8}$
$\frac{27}{8} = 3\frac{3}{8}$

SOLUTION.—Reducing $4\frac{1}{2}$ and $1\frac{1}{3}$ to improper fractions (§ **104**), we have $\frac{9}{2}$ and $\frac{4}{3}$. Dividing $\frac{9}{2}$ by $\frac{4}{3}$, the result is $3\frac{3}{8}$.

Divide:

20.	$2\frac{2}{5}$ by 6.	$\frac{2}{5}$.	**28.**	$2\frac{1}{4}$ by $7\frac{1}{2}$.	$\frac{3}{10}$.
21.	22 by $5\frac{1}{2}$.	4.	**29.**	$3\frac{2}{3}$ by 7.	$\frac{11}{21}$.
22.	$2\frac{1}{2}$ by $\frac{1}{16}$.	40.	**30.**	50 by $4\frac{3}{7}$.	$11\frac{9}{31}$.
23.	$4\frac{4}{5}$ by 8.	$\frac{3}{5}$.	**31.**	$\frac{1}{2}$ by $\frac{1}{50}$.	25.
24.	6 by $2\frac{2}{5}$.	$2\frac{1}{2}$.	**32.**	$47\frac{2}{5}$ by 15.	$3\frac{4}{25}$.
25.	$4\frac{3}{4}$ by $5\frac{1}{8}$.	$\frac{38}{41}$.	**33.**	56 by $5\frac{4}{9}$.	$10\frac{2}{7}$.
26.	$12\frac{4}{7}$ by 11.	$1\frac{1}{7}$.	**34.**	$\frac{14}{15}$ by 21.	$\frac{2}{45}$.
27.	30 by $3\frac{3}{4}$.	8.	**35.**	$130\frac{2}{3}$ by 18.	$7\frac{7}{27}$.

36. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{4}{5}$.

OPERATION

$$\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3} \times \frac{\cancel{4}}{3} \times \frac{5}{\cancel{4}} = \frac{5}{9}$$

EXPLANATION.—Invert the terms of both $\frac{3}{4}$ and $\frac{4}{5}$.

37. Divide $\frac{3}{5}$ of $\frac{8}{9}$ by $\frac{6}{7}$ of $\frac{3}{4}$. $\frac{112}{135}$.

38. Divide $\frac{1}{3}$ of $5\frac{1}{8}$ by $\frac{3}{4}$ of $17\frac{1}{2}$. $\frac{41}{315}$.

39. Divide $\frac{5}{18}$ of $\frac{2}{5}$ of $12\frac{3}{10}$ by $\frac{1}{5}$ of $8\frac{1}{5}$. $\frac{5}{6}$.

40. Divide $\frac{2}{7}$ of $\frac{7}{8}$ by $\frac{3}{4}$ of $\frac{1}{3}$ of 5. $\frac{1}{5}$.

41. Divide $\frac{5}{18}$ of $\frac{2}{5}$ of $12\frac{3}{10}$ by $\frac{1}{5}$ of $4\frac{1}{10}$ of 20. $\frac{1}{12}$.

120. To find what part a smaller number is of a larger number:

1. 1 is what part of 2?

SOLUTION. — 1 is $\frac{1}{2}$ of 2; for $\frac{1}{2}$ of 2 is $\frac{2}{2}$, or 1 (§ **96**).

OPERATION
$$\frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$$

2. 2 is what part of 3?

SOLUTION. — 1 is $\frac{1}{3}$ of 3; then, 2 is 2 times $\frac{1}{3} = \frac{2}{3}$ of 3.

OPERATION
$$\frac{2}{1} \times \frac{1}{3} = \frac{2}{3}$$

3. $\frac{1}{2}$ is what part of 3?

SOLUTION. — 1 is $\frac{1}{3}$ of 3; then, $\frac{1}{2}$ is $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ of 3.

OPERATION
$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

4. $\frac{2}{3}$ is what part of $\frac{3}{4}$?

SOLUTION. — $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, and $\frac{4}{4}$, or 1, is 4 times $\frac{1}{3} = \frac{4}{3}$ of $\frac{3}{4}$; then, $\frac{1}{3}$ is $\frac{1}{3}$ of $\frac{4}{3} = \frac{4}{9}$ of $\frac{3}{4}$, and $\frac{2}{3}$ is 2 times $\frac{4}{9} = \frac{8}{9}$ of $\frac{3}{4}$.

OPERATION
$$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

Rule. — *Divide the smaller number by the larger number.*

5. 3 is what part of 4? $\frac{3}{4}$.

6. $\frac{3}{4}$ is what part of 5? $\frac{3}{20}$.

7. $\frac{1}{4}$ is what part of $\frac{1}{2}$? $\frac{1}{2}$.

8. $\frac{2}{3}$ is what part of $\frac{5}{6}$? $\frac{4}{5}$.

9. $3\frac{3}{4}$ is what part of 5? $\frac{3}{4}$.

10. $\frac{5}{6}$ is what part of $\frac{8}{9}$? $\frac{15}{16}$.

11. $8\frac{5}{9}$ is what part of 11? $\frac{7}{9}$.

12. $\frac{21}{32}$ is what part of $\frac{35}{48}$? $\frac{9}{10}$.

13. $1\frac{1}{4}$ is what part of $2\frac{1}{3}$?

SOLUTION. — Reducing $1\frac{1}{4}$ and $2\frac{1}{3}$ to improper fractions (§ **104**), we have $\frac{5}{4}$ and $\frac{7}{3}$. Dividing $\frac{5}{4}$ by $\frac{7}{3}$ (§ **119**), the result is $\frac{15}{28}$.

OPERATION
$$1\frac{1}{4} = \frac{5}{4}$$
$$2\frac{1}{3} = \frac{7}{3}$$
$$\frac{5}{4} \times \frac{3}{7} = \frac{15}{28}$$

NOTE. — Problem 13 might be expressed. Find the value of $\dfrac{1\frac{1}{4}}{2\frac{1}{3}}$. A fraction that has a fraction in one or both of its terms is called a complex fraction.

Find the value of:

14. $\dfrac{\frac{6}{7}}{\frac{11}{5}}$.	$\frac{30}{77}$.	16. $\dfrac{2}{3\frac{2}{3}}$.	$\frac{6}{11}$.	18. $\dfrac{2\frac{1}{3}}{4\frac{1}{2}}$.	$\frac{14}{27}$.
15. $\dfrac{\frac{2}{3}}{5}$.	$\frac{2}{15}$.	17. $\dfrac{3\frac{1}{8}}{4\frac{5}{7}}$.	$\frac{175}{264}$.	19. $\dfrac{3\frac{3}{4}}{5\frac{5}{8}}$.	$\frac{2}{3}$.

121. To find a number when the value of a fractional part of it is given:

1. 150 is $\frac{3}{4}$ of what number?

OPERATION

$$150 \div \frac{3}{4} =$$

$$\overset{50}{\cancel{150}} \times \frac{4}{\cancel{3}} = 200$$

SOLUTION. — Since 150 is $\frac{3}{4}$ of a number, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 150, or 50; and since 50 is $\frac{1}{4}$ of the number, the number must be 4 times 50, or 200. Hence, 150 is $\frac{3}{4}$ of 200.

Rule. — *Divide the given value by the fractional part.*

2. 120 is $\frac{4}{5}$ of what number? 150.
3. 180 is $\frac{3}{4}$ of what number? 240.
4. 392 is $\frac{4}{9}$ of what number? 882.
5. 418 is $\frac{11}{12}$ of what number? 456.
6. 625 is $\frac{25}{32}$ of what number? 800.
7. 136 is $\frac{8}{13}$ of what number? 221.

ALIQUOT PARTS OF 100

122. An **aliquot part** is an exact divisor of a number.

$50 = \frac{1}{2}$ of 100	$14\frac{2}{7} = \frac{1}{7}$ of 100	$8\frac{1}{3} = \frac{1}{12}$ of 100
$33\frac{1}{3} = \frac{1}{3}$ of 100	$12\frac{1}{2} = \frac{1}{8}$ of 100	$6\frac{2}{3} = \frac{1}{15}$ of 100
$25 = \frac{1}{4}$ of 100	$11\frac{1}{9} = \frac{1}{9}$ of 100	$6\frac{1}{4} = \frac{1}{16}$ of 100
$20 = \frac{1}{5}$ of 100	$10 = \frac{1}{10}$ of 100	$5 = \frac{1}{20}$ of 100
$16\frac{2}{3} = \frac{1}{6}$ of 100	$9\frac{1}{11} = \frac{1}{11}$ of 100	$2 = \frac{1}{50}$ of 100

The following multiples of aliquot parts of 100 are often used:

$66\frac{2}{3} = \frac{2}{3}$ of 100	$83\frac{1}{3} = \frac{5}{6}$ of 100
$75 = \frac{3}{4}$ of 100	$37\frac{1}{2} = \frac{3}{8}$ of 100
$40 = \frac{2}{5}$ of 100	$62\frac{1}{2} = \frac{5}{8}$ of 100
$60 = \frac{3}{5}$ of 100	$87\frac{1}{2} = \frac{7}{8}$ of 100
$80 = \frac{4}{5}$ of 100	$18\frac{3}{4} = \frac{3}{16}$ of 100

1. Find the cost of 24 yd. of lace at 25¢ a yard.

OPERATION

SOLUTION. — Since 25¢ is $\frac{1}{4}$ of a dollar, the cost will be $\frac{1}{4}$ as many dollars as there are yards. $\frac{1}{4}$ of $24 is $6.

$4\overline{)24}$
$6

2. I spent $1.25 for ribbon at $12\frac{1}{2}$¢ a yard. How many yards did I buy?

OPERATION

SOLUTION. — Since $12\frac{1}{2}$¢ is $\frac{1}{8}$ of a dollar, 8 yards can be bought for one dollar, and there will be 8 times as many yards as there are dollars. $8 \times 1\frac{1}{4} = 10$ yd.

$1\frac{1}{4}$
$\underline{8}$
10 yd.

3. Find the cost of 20 yd. of ribbon at $18\frac{3}{4}$¢ a yard. $3.75.

4. Multiply $3112 by $87\frac{1}{2}$.

OPERATION

SOLUTION. — Since $87\frac{1}{2}$ is $\frac{7}{8}$ of 100, multiply 3112 by 100 and multiply the product by $\frac{7}{8}$.

$3112 \times 100 = 311200$

$\frac{7}{\cancel{8}}$ of $\overset{38900}{\cancel{311200}} = 272300$

5. Divide 500 by $83\frac{1}{3}$.

OPERATION

SOLUTION. — Since $83\frac{1}{3}$ is $\frac{5}{6}$ of 100, divide 500 by 100 and divide the quotient by $\frac{5}{6}$.

$500 \div 100 = 5$

$5 \div \frac{5}{6} = \cancel{5} \times \frac{6}{\cancel{5}} = 6$

Multiply:

6. 8000 by $87\frac{1}{2}$. 700,000.
7. 3600 by $83\frac{1}{3}$. 300,000.
8. 6474 by $66\frac{2}{3}$. 431,600.

Divide:

9. 6000 by 75. 80.
10. 7500 by $16\frac{2}{3}$. 450.
11. 2000 by $6\frac{1}{4}$. 320.

12. If I pay \$2.25 for lace at $18\frac{3}{4}$¢ a yard, how many yards do I buy? 12 yd.

13. How much will 6 yd. of linen cost at \$0.62$\frac{1}{2}$ a yard? \$3.75.

14. If I pay \$66.25 for books at \$3.75 a dozen, how many dozen books do I buy? $17\frac{2}{3}$ doz.

15. How much will $6\frac{2}{3}$ doz. handkerchiefs cost at $37\frac{1}{2}$¢ apiece? \$30.

16. How many times is \$4.87$\frac{1}{2}$ contained in \$39? 8.

17. How much will 36 neckties cost at $33\frac{1}{3}$¢ apiece? \$12.

18. How much will $3\frac{1}{2}$ yd. of lace cost at \$1.75 a yard? \$6.12$\frac{1}{2}$.

19. At \$1.50 a yard, how much cloth can be bought for \$7.12$\frac{1}{2}$? $4\frac{3}{4}$ yd.

20. What is the cost of 100 readers at \$3.90 a dozen? \$32.50.

21. What is the cost of $3\frac{3}{4}$ doz. knives at \$5.40 a dozen? \$20.25.

FRACTIONAL COMPOUND DENOMINATE NUMBERS

123. 1. Add $16\frac{1}{16}$ yd.; $9\frac{1}{8}$ yd.; $5\frac{7}{16}$ yd.; $2\frac{13}{16}$ yd. $33\frac{7}{16}$ yd.

2. I paid for books \$9$\frac{1}{4}$; for a fountain pen, \$4$\frac{1}{2}$; and for stationery, \$1$\frac{1}{4}$. What amount did I expend? \$15.

3. Having \$50$\frac{1}{4}$, I paid a bill of \$27$\frac{1}{2}$. How much had I left? \$22$\frac{3}{4}$.

4. From \$32.31$\frac{1}{4}$ take \$15.12$\frac{1}{2}$. \$17.18$\frac{3}{4}$.

5. From \$5.81$\frac{1}{4}$ take \$1.18$\frac{3}{4}$. \$4.62$\frac{1}{2}$.

Find the cost of:

6. 9 yd. of dimity, at $12\frac{1}{2}$¢ a yard. 1.12\frac{1}{2}$.

7. 21 lb. of sugar, at $6\frac{1}{4}$¢ a pound. 1.31\frac{1}{4}$.

8. 15 yd. of cloth, at $$3\frac{1}{4}$ per yard. $48.75.

9. $5\frac{1}{2}$ yd. of linen, at $$\frac{1}{2}$ per yard. $2.75.

10. $12\frac{1}{2}$ yd. of ribbon, at $12\frac{1}{2}$¢ per yard. 1.56\frac{1}{4}$.

11. $13\frac{1}{2}$ yd. of lawn, at $12\frac{1}{2}$¢ per yard. 1.68\frac{3}{4}$.

12. $10\frac{1}{4}$ yd. of cloth, at 3.37\frac{1}{2}$ a yard. 34.59\frac{3}{8}$.

13. $17\frac{2}{3}$ doz. books, at $3.75 per dozen. $66.25.

14. At $13\frac{1}{2}$¢ per yard, how many yards of gingham can be purchased for $2.70? 20 yd.

15. At $37\frac{1}{2}$¢ a yard, how many yards of lawn can you buy for 5.81\frac{1}{4}$? $15\frac{1}{2}$ yd.

16. If 5 yd. of cloth cost 11\frac{1}{4}$, what is the cost of one yard? 2\frac{1}{4}$.

17. A line $31\frac{1}{16}$ in. long is divided into 7 parts. How long is each part? $4\frac{7}{16}$ in.

18. Reduce 5 mi. to inches. 316800 in.

19. Reduce 2 mi. 2 rd. 2 ft. to feet. 10595 ft.

20. Reduce 20 yd. to rods. 3 rd. $3\frac{1}{2}$ yd.

21. Reduce 15875 ft. to miles. 3 mi. 2 rd. 2 ft.

22. Reduce 142634 in. to miles. 2 mi. 80 rd. 2 yd. 2 in.

23. How many steps, of 2 ft. 8 in. each, will a man take in walking 2 miles? 3960.

24. How many revolutions will a wheel, of 9 ft. 2 in. circumference, make in running 65 miles? 37440.

25. Reduce 1 A. 136 sq. rd. 25 sq. yd. to square yards. 8979 sq. yd.

26. Reduce 7506 sq. yd. to A. 1 A. 88 sq. rd. 4 sq. yd.

27. Reduce 5 chains 15 links to in. 4078$\frac{4}{5}$ in.

28. How many acres are there in a field 40$\frac{1}{2}$ rd. long and 32 rd. wide? 8 A. 16 sq. rd.

29. Reduce 4 yr. to hours. 35064 hr.

30. Reduce 914092 hr. to centuries.
1 cen. 4 yr. 101 da. 4 hr.

31. In what time will a body move from the earth to the moon, at the rate of 31 miles per day, the distance being 238545 miles? 21 yr. 24$\frac{3}{4}$ da.

124. A fraction is reduced to a lower denomination by multiplication (§ **63**, Rule I).

1. Reduce $\frac{1}{24}$ of a peck to the fraction of a pint.

SOLUTION. — To reduce $\frac{1}{24}$ of a peck to the fraction of a pint, multiply by 8 and by 2, since there are 8 qt. in 1 pk. and 2 pt. in 1 qt. The result is $\frac{2}{3}$ of a pint.

OPERATION

$$\frac{\cancel{8}}{1}\times\frac{2}{1}\times\frac{1}{\underset{3}{\cancel{24}}}=\frac{2}{3}$$

Reduce:

2. $\frac{1}{40}$ bu. to quarts. $\frac{4}{5}$ qt. **4.** $\frac{1}{20}$ rd. to feet. $\frac{33}{40}$ ft.

3. $\frac{1}{28}$ lb. to ounces. $\frac{4}{7}$ oz. **5.** $\frac{1}{1584}$ da. to min. $\frac{10}{11}$ min.

125. In reducing a fraction to a lower denomination, when the result is a mixed number, proceed only with the reduction of the fractional part. This is called finding the value of a fraction *in integers*.

1. Find the value of $\frac{2}{5}$ of a day in integers.

SOLUTION. — To reduce $\frac{2}{5}$ of a day to hours, multiply by 24; the result is 9$\frac{3}{5}$ hr. To reduce $\frac{3}{5}$ of an hour to minutes, multiply by 60; the result is 36 min. $\frac{2}{5}$ of a day, then, is 9 hr. 36 min.

OPERATION

$\frac{24}{1}\times\frac{2}{5}=9\frac{3}{5}$

$\frac{60}{1}\times\frac{3}{5}=36$

2. Find the value of $\frac{4}{5}$ mi. in integers. 256 rd.

3. Find the value of \$$\frac{3}{5}$ in integers. 60¢.

4. Find the value of $\frac{4}{5}$ lb. Troy in integers. 9 oz. 12 pwt.

126. A fraction is reduced to a higher denomination by division (§ **63**, Rule II).

1. Reduce $\frac{2}{3}$ of a pint to the fraction of a peck.

SOLUTION. — To reduce $\frac{2}{3}$ of a pint to the fraction of a peck, divide by 2 and by 8, since 2 pt. make 1 qt. and 8 qt. make 1 pk. The result is $\frac{1}{24}$ of a peck.

OPERATION

$$\frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} \times \frac{1}{8} = \frac{1}{24}$$

Reduce :

2. $\frac{4}{5}$ ft. to rods. $\frac{8}{165}$.

3. $\frac{3}{80}$ oz. (Av.) to lbs. $\frac{3}{1280}$.

4. $\frac{3}{4}$ in. to rods. $\frac{1}{264}$.

5. $\frac{8}{9}$ min. to days. $\frac{1}{1620}$.

127. To find what part one compound denominate number is of another, reduce them to the same denomination and proceed as in § **120**.

1. 2 ft. 3 in. is what part of a yard?

SOLUTION. — 2 ft. 3 in. equals 27 in. 1 yd. equals 36 in. 27 in. are $\frac{27}{36}$ of 36 in. $\frac{27}{36}$ equals $\frac{3}{4}$. 2 ft. 3 in. then is $\frac{3}{4}$ of a yard.

OPERATION

2 ft. 3 in. = 27 in.
1 yd. = 36 in.
$\frac{27}{36} = \frac{3}{4}$

2. 2 ft. 6 in. is what part of 6 ft. 8 in.? $\frac{3}{8}$.

3. 2 pk. 4 qt. is what part of a bushel? $\frac{5}{8}$.

4. What part is 2 yd. 9 in. of 8 yd. 2 ft. 3 in.? $\frac{9}{35}$.

5. What part of a day is 13 hr. 30 min.? $\frac{9}{16}$.

6. 15 mi. 123 rd. is what part of 35 mi. 287 rd.? $\frac{3}{7}$.

128. To add and subtract fractional compound numbers, find the value of the fractions in integers, and then proceed as in addition and subtraction of compound denominate numbers.

1. Add $\frac{3}{4}$ yd. and $\frac{5}{6}$ ft.

SOLUTION. — $\frac{3}{4}$ yd. equals 2 ft. 3 in.; $\frac{5}{6}$ ft. equals 10 in.; the sum of 2 ft. 3 in. and 10 in. is 3 ft. 1 in. (§ **75**).

OPERATION

$\frac{3}{4}$ yd. = 2 ft. 3 in.
$\frac{5}{6}$ ft. = 10 in.
3 ft. 1 in.

2. From $\frac{2}{9}$ da. subtract $\frac{5}{6}$ hr.

SOLUTION. — $\frac{2}{9}$ da. equals 5 hr. 20 min.; $\frac{5}{6}$ hr. equals 50 min.; 50 min. subtracted from 5 hr. 20 min. leaves 4 hr. 30 min. (§ **76**).

OPERATION

$\frac{2}{9}$ da. = 5 hr. 20 min.
$\frac{5}{6}$ hr. = 50 min.
4 hr. 30 min.

3. Add $\frac{2}{3}$ da. and $\frac{3}{4}$ hr. 16 hr. 45 min.

4. Add $\frac{2}{3}$ wk. $\frac{5}{9}$ da. $\frac{2}{3}$ hr. and $\frac{2}{3}$ min. 5 da. 6 hr. 40 sec.

5. Add $\frac{11}{12}$ gal. and $\frac{1}{12}$ qt. 3 qt. 1 pt. 2 gi.

6. From $\frac{7}{9}$ da. subtract $\frac{1}{18}$ hr. 18 hr. 36 min. 40 sec.

7. From $\frac{3}{8}$ lb. subtract $\frac{7}{8}$ oz. $5\frac{1}{8}$ oz.

8. From $\frac{1}{7}$ da. subtract $\frac{6}{7}$ hr. 2 hr. 34 min. $17\frac{1}{7}$ sec.

Miscellaneous Examples

129. **1.** Reduce $\frac{32989}{56981}$ to its lowest terms. $\frac{11}{19}$.

2. Add $\frac{5}{14}$, $\frac{8}{21}$, $2\frac{1}{2}$, $3\frac{2}{3}$. $6\frac{19}{21}$.

3. From $3\frac{4}{7}$ subtract $1\frac{4}{5}$. $1\frac{27}{35}$.

4. From $3\frac{5}{8}$ subtract $\frac{1}{3}$ of $3\frac{1}{2}$. $2\frac{11}{24}$.

5. Add $\frac{5}{9}$ of $\frac{7}{10}$ and $\frac{2}{5}$ of $\frac{7}{12}$. $\frac{28}{45}$.

6. Add $1\frac{3}{4} \div 2\frac{1}{2}$ and $5\frac{1}{2} \div 3\frac{1}{8}$. $2\frac{23}{50}$.

7. What number divided by $\frac{3}{5}$ will give 10 for a quotient? 6.

8. What number multiplied by $\frac{3}{5}$ will give 10 for a product? $16\frac{2}{3}$.

9. What number is that from which if you take $\frac{3}{7}$ of itself, the remainder will be 16? 28.

10. What number is that to which if you add $\frac{3}{7}$ of itself, the sum will be 20? 14.

11. A boat is worth $900; a merchant owns $\frac{5}{8}$ of it, and sells $\frac{1}{3}$ of his share. What part has he left, and what is it worth? $\frac{5}{12}$ left, worth $375.

12. I own $\frac{7}{12}$ of a ship, and sell $\frac{1}{3}$ of my share for 1944\frac{4}{9}$. What is the whole ship worth? $10000.

13. What part of 3 cents is $\frac{2}{3}$ of 2 cents? $\frac{4}{9}$.

14. What part of 368 is 176? $\frac{11}{23}$.

15. From $\frac{25}{37}$ subtract the sum of $\frac{1}{8}$, $\frac{1}{18}$, and $\frac{13}{111}$. $\frac{1007}{2664}$.

16. From 1 subtract $\frac{3}{10}$ of $\frac{7}{12}$ of $4\frac{9}{14}$. $\frac{3}{16}$.

17. From $\frac{2}{3} \div \frac{5}{7}$ subtract $\frac{5}{8} \div \frac{10}{11}$. $\frac{59}{240}$.

18. If I ride 2044 rods in $\frac{7}{15}$ hr., at that rate how far will I ride in $1\frac{14}{15}$ hr.? 8468 rd.

19. What part of $1\frac{1}{4}$ ft. are $3\frac{1}{3}$ in.? $\frac{2}{9}$.

20. Two men bought a barrel of fine flour; one paid 3\frac{1}{5}$ and the other 3\frac{2}{3}$. What part of it should each have? One $\frac{48}{103}$, the other $\frac{55}{103}$.

21. A has $2400; $\frac{5}{8}$ of his money + $500 is $\frac{5}{4}$ of B's. What sum has B? $1600.

22. Gordon Apgar divided his estate among 2 sons and 3 daughters, the latter sharing equally with each other. The younger son received $2200, which was $\frac{5}{12}$ of the share of the elder, whose share was $\frac{16}{35}$ of the whole estate. Find the share of each daughter. 1356\frac{2}{3}$.

DECIMAL FRACTIONS

130. The orders of integers decrease from left to right in a *tenfold* ratio.

Thus, in the number 1111, 1 thousand is 10 times 1 hundred, 1 hundred is 10 times 1 ten, and 1 ten is 10 times 1 unit.

Orders of Decimals

131. The orders may be continued from the order of units toward the right by the same law of decrease.

Let the order units be separated from the order that follows by a point (.).

Then, in the number 1.111,

Since the 1 to the left of the point is 1 unit, the 1 to the right of the point is 1 tenth; for 1 unit is 10 times $\frac{1}{10}$.

Since the first order from the unit is 1 tenth, the second order from the unit is 1 hundredth; for $\frac{1}{10}$ is 10 times $\frac{1}{100}$.

Since the second order from the unit is 1 hundredth, the third order from the unit is 1 thousandth; for $\frac{1}{100}$ is 10 times $\frac{1}{1000}$.

In like manner it may be shown that one in the fourth order to the right from the unit is 1 ten-thousandth; 1 in the fifth order to the right is 1 hundred-thousandth; 1 in the sixth order is 1 millionth, etc.

NOTE. — A number consisting of figures other than 1 might be used as well for the purpose of illustration.

132. The relation of decimals and integers to each other is shown in the following table:

Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units		Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths
8	9	4	3	7	2	8	.	2	9	4	3	2	7	8

The first order on the *left* of the unit is *tens;* the first order on the *right* of the unit is *tenths;* the second order on the *left* is *hundreds;* the second order on the *right* is *hundredths*, etc.

133. A **decimal fraction,** or *decimal*, is one or more *tenths*, *hundredths*, *thousandths*, etc., written like the orders of integers.

A **decimal point** (.) is placed before the order tenths to distinguish the fraction.

The decimal orders increase from right to left, and decrease from left to right, the same as the orders of integers.

The names of the orders of decimals are similar to the names of the corresponding orders of integers.

134. Conversion of the common fractions $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, etc., to decimals:

1. $\frac{1}{10}$ is written .1

$\frac{2}{10}$ are written .2

$\frac{3}{10}$ are written .3

$\frac{4}{10}$ are written .4

$\frac{5}{10}$ are written .5

$\frac{6}{10}$ are written .6

$\frac{7}{10}$ are written .7

$\frac{8}{10}$ are written .8

$\frac{9}{10}$ are written .9

When the denominator is 10, *there is one decimal order.*

2. $\frac{1}{100}$ is written .01; there being no tenths, a cipher is written in the vacant order.

$\frac{2}{100}$ are written .02	$\frac{6}{100}$ are written .06
$\frac{3}{100}$ are written .03	$\frac{7}{100}$ are written .07
$\frac{4}{100}$ are written .04	$\frac{8}{100}$ are written .08
$\frac{5}{100}$ are written .05	$\frac{9}{100}$ are written .09

When the denominator is 100, *there are two decimal orders.*

3. $\frac{1}{1000}$ is written .001; there being no tenths and no hundredths, ciphers are written in the vacant orders.

$\frac{2}{1000}$ are written .002	$\frac{6}{1000}$ are written .006
$\frac{3}{1000}$ are written .003	$\frac{7}{1000}$ are written .007
$\frac{4}{1000}$ are written .004	$\frac{8}{1000}$ are written .008
$\frac{5}{1000}$ are written .005	$\frac{9}{1000}$ are written .009

When the denominator is 1000, *there are three decimal orders.*

4. In like manner:

$\frac{1}{10000}$ is written .0001
$\frac{1}{100000}$ is written .00001
$\frac{1}{1000000}$ is written .000001

The number of orders in the decimal is always the same as the number of ciphers in the denominator of the common fraction.

5. $\frac{1}{10}$ and $\frac{1}{100}$ are $\frac{11}{100}$, written .11.
$\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$ are $\frac{111}{1000}$, written .111.
$\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$ are $\frac{1111}{10000}$, written .1111.

Hence, tenths and hundredths are read as *hundredths*; tenths, hundredths, and thousandths are read as *thousandths;* tenths, hundredths, thousandths, and ten-thousandths are read as *ten-thousandths*, etc.

The **numerator** of a decimal is the number it expresses disregarding the decimal point.

If there are vacant orders before the numerator, ciphers are written in them.

The name of the right-hand order is the name of the decimal.

135. To write decimals:

1. Write two hundred sixty-five *thousandths*.

Explanation. — First, write the numerator, 265, as an integer. The figure 5 must stand in the order thousandths; then, 6 must be hundredths and 2 must be tenths; the decimal point, therefore, is placed before the figure 2.

.265

2. Write two hundred sixty-five *millionths*.

Explanation. — Write the numerator, 265, as an integer. The figure 5 must stand in the order millionths; then, 6 must be hundred-thousandths, 2 must be ten-thousandths, and ciphers must be written in the orders thousandths, hundredths, and tenths; the decimal point is placed before 0 tenths.

.000265

3. Write two hundred sixty-five *hundredths*.

Explanation. — Write the numerator, 265, as an integer. The figure 5 must stand in the order hundredths; then, 6 must be tenths; the decimal point, therefore, is placed between the figures 2 and 6.

2.65

Note. — This number should not be read two hundred and sixty-five hundredths. The word *and* should never be used except between integers and decimals. Thus, 200.65 is read two hundred *and* sixty five hundredths.

4. Write four hundred ninety-eight and two hundred sixty-five *millionths.*

EXPLANATION.—First write the decimal as in Ex. 2; then write the integer, placing it at the left of the decimal point.

498.000265

Rule.—1. *Write the numerator as an integer.*

2. *Place the decimal point so that the name of the right-hand order shall be the same as the name of the decimal.*

NOTES.—1. Pupils should be rendered familiar with the decimal orders so as to name them readily, in succession, both from left to right, and from right to left.

2. When the decimal is a proper fraction it is sometimes necessary to prefix ciphers to the numerator (Ex. 2).

3. When the decimal is an improper fraction the decimal point is placed between two of the figures of the numerator (Ex. 3).

4. In a mixed number the decimal point is placed after the units' order of the integer (Ex. 4).

Write the following decimal numbers:

5. Twenty-six *hundredths.* .26

6. Thirty-five *hundredths.*

7. Eighty-seven *hundredths.*

8. Four hundred nineteen *hundredths.*

9. Five *thousandths.*

10. Fifty-four *thousandths.*

11. Three hundred four *thousandths.*

12. Seven thousand two hundred ninety-three *thousandths.*

13. Twenty-five, and forty-seven *thousandths.*

14. Two hundred five *ten-thousandths.*

15. Four thousand one hundred twenty-five *ten-thousandths.*

16. Nine *hundred-thousandths.*

17. Nine hundred *thousandths.*

18. Six hundred five *hundred-thousandths.*

19. Twenty thousand three hundred four *hundred-thousandths.*

20. Seven *millionths.*

21. Two hundred three *millionths.*

22. Three hundred thousand four *millionths.*

23. Twenty-four *ten-millionths.*

24. Eighty thousand six *ten-millionths.*

25. Two hundred *millionths.*

26. Two *hundred-millionths.*

27. Nine hundred seven *hundred-millionths.*

28. Twenty million twenty thousand three *hundred-millionths.*

29. One million ten thousand one hundred *millionths.*

30. One million ten thousand one *hundred-millionths.*

31. One hundred six, and thirty-seven *thousandths.*

32. One thousand, and one *thousandth.*

33. Two hundred twenty-five *thousandths.*

34. Two hundred, and twenty-five *thousandths.*

35. Two thousand nine hundred twenty-nine *millionths.*

36. Two thousand nine hundred, and twenty-nine *millionths.*

37. One million five *billionths.*

38. Two hundred two *ten-billionths.*

39. Two hundred, and two *ten-billionths.*

40. Sixty-five, and six thousand five *millionths.*

Change the following common fractions to decimals:

41. $\frac{3}{10}$, $\frac{7}{10}$, $\frac{9}{100}$, $\frac{17}{100}$, $\frac{23}{100}$, $\frac{41}{100}$, $\frac{53}{100}$. $\frac{3}{10} = .3$

42. $\frac{87}{100}$, $\frac{97}{100}$, $\frac{123}{1000}$, $\frac{289}{1000}$, $\frac{487}{1000}$, $\frac{733}{1000}$.

43. $\frac{3}{1000}$, $\frac{101}{10000}$, $\frac{53}{100000}$, $\frac{503}{1000000}$. $\frac{9}{100} = .09$

136. To read decimals:

1. Read .265.

NUMBER READ. — Two hundred sixty-five thousandths.

EXPLANATION. — Disregarding the decimal point, the number is two hundred sixty-five; this is the numerator of the decimal. The right-hand order of the decimal is thousandths; this is the name of the decimal.

2. Read .000265.

NUMBER READ. — Two hundred sixty-five millionths.

EXPLANATION. — Disregarding the decimal point, the number is two hundred sixty-five; this is the numerator of the decimal. The right hand order is millionths; this is the name of the decimal.

3. Read 2.65.

NUMBER READ. — Two, and sixty-five hundredths, or two hundred sixty-five hundredths.

Rule. — 1. *Disregarding the decimal point, read the number as an integer.*

2. *Give the name of the right-hand order.*

NOTES. — 1. Before commencing to read the decimal, the name of the right-hand order should be ascertained.

2. A mixed number may be read either as an integer and a fraction, or as an improper fraction (Ex. 3).

Read the following decimal numbers:

4. .028, .341, 2.327, 50.005, 184.173.

5. .0003, .0625, .2374, .2006, .0104.

6. 3.0205, 810.2406, 10720.0905.

7. .00004, .00137, .02376, .01007.
8. .001768, .040035, 70.360004.
9. .1010101, .00040005, .00100304.
10. .31456, .000133, 60.04, 45.1003.
11. 357.75, .4928, 5.945, 681.0002.
12. 70.1200764, 954.203, 38.027.
13. 1007.3154, 7496.35491768.
14. .00715, 3.00005, 28.10065701.
15. 13.008241.

Change the following decimals to common fractions:

16. .9, .13, .19, .29, .37, .73.
17. .91, .347, .513, .691, .851, .917.
18. .007, .0207, .00079, .001007.
19. 1.36, .3421, .03401, .0900.
20. .001, .5302, 8.01, .000053.

137. The operations with decimals are *reduction, addition, subtraction, multiplication, and division.*

REDUCTION OF DECIMALS

138. Reduction of decimals is changing their form without altering their value.

139. Principles. — I. *Annexing decimal ciphers to an integer does not change its value.*

Thus, 7.00 is the same as 7; for 7.00 is 7 and no hundredths (§ **136**, Rule). This corresponds to § **103**.

II. *Omitting decimal ciphers from the right of an integer does not change its value.*

This corresponds to § **105**.

140. Principles. — I. *Annexing ciphers to a decimal does not change its value.*

Thus, .70 is the same as .7; for $\frac{7}{10}=\frac{70}{100}$. This corresponds to § **106**.

II. *Omitting ciphers from the right of a decimal does not change its value.*

This corresponds to § **107**.

141. To reduce a decimal to a common fraction:

1. Reduce .75 to a common fraction.

SOLUTION. — 75 hundredths written as a common fraction is $\frac{75}{100}$. $\frac{75}{100}$ reduced to its lowest terms (§ **107**) is $\frac{3}{4}$.

OPERATION

$.75 = \frac{75}{100}$

$\frac{75}{100} = \frac{3}{4}$

Rule. — 1. *Write the decimal as a common fraction.*
2. *Reduce the fraction to its lowest terms.*

Reduce to common fractions:

2. .6.	$\frac{3}{5}$.	5. .035.	$\frac{7}{200}$.
3. .25.	$\frac{1}{4}$.	6. .5625.	$\frac{9}{16}$.
4. .375.	$\frac{3}{8}$.	7. .34375.	$\frac{11}{32}$.

8. Express 4.02 as an integer and common fraction. $4\frac{1}{50}$.

9. Express 8.415 as an integer and common fraction. $8\frac{83}{200}$.

142. To reduce a common fraction to a decimal:

1. Reduce $\frac{3}{4}$ to a decimal.

SOLUTION. — Annexing a decimal cipher to 3, it is 3.0; 30 tenths divided by 4 is 7 tenths, and 2 tenths remaining. Annexing a cipher to .2 it is .20; 20 hundredths divided by 4 is 5 hundredths. The result is .75.

OPERATION

4)3.00
.75

EXPLANATION. — $\frac{3}{4}$ is 3 divided by 4 (§ **97**). Annexing a decimal cipher to 3 does not change its value (§ **139**). Annexing a cipher to .2 does not change its value (§ **140**).

Rule. — 1. *Annex decimal ciphers to the numerator.*

2. *Divide by the denominator.*

3. *Point off as many decimal orders in the quotient as there are decimal ciphers annexed to the numerator.*

Reduce to decimals:

2.	$\frac{4}{5}$.	.8.	**7.**	$\frac{1}{1250}$.	.0008.
3.	$\frac{5}{8}$.	.625.	**8.**	$\frac{9}{400}$.	.0225.
4.	$\frac{7}{25}$.	.28.	**9.**	$\frac{1}{256}$.	.00390625.
5.	$\frac{3}{40}$.	.075.	**10.**	$\frac{5}{6}$.	.83+.
6.	$\frac{15}{16}$.	.9375.	**11.**	$\frac{1}{11}$.	.09+.

ADDITION OF DECIMALS

143. Addition of decimals is the process of finding the sum of two or more decimal numbers.

1. Add 375.83; 49.627; 5842.1963; 813.9762.

SOLUTION. — Write the numbers so that the four decimal points may be in a column, the units 5, 9, 2, 3 in the first column to the left, the tenths 8, 6, 1, 9 in the first column to the right, etc.; then, adding as in integers, place the decimal point in the sum between 1 and 6 under the column of decimal points.

OPERATION

$$\begin{array}{r} 375.83 \\ 49.627 \\ 5842.1963 \\ 813.9762 \\ \hline 7081.6295 \end{array}$$

Rule. — 1. *Write the numbers so that the decimal points and figures of the same order may stand in the same column.*

2. *Add as in integers.*

3. *Place the decimal point in the sum under the column of decimal points.*

2. Add 37.1065; 432.07; 4.20733; 11.706. 485.08983.

3. Find the sum of 3, and 25 hundredths; 6, and 4 tenths; and 35 hundredths. 10.

4. Add 4, and 4 ten-thousandths; 28, and 35 thousandths; 8, and 7 hundredths; and 9404 hundred-thousandths. 40.19944.

5. Add 21.611; 6888.32; 3.4167. 6913.3477.

6. Add 6.61; 636.1; 6516.14; 67.1234; and 5.1233. 7231.0967.

7. Add 4, and 8 tenths; 43, and 31 hundredths; 74, and 19 thousandths; 11, and 204 thousandths. 133.333.

8. Add 45, and 19 thousandths; 7, and 71 hundred-thousandths; 93, and 4327 ten-thousandths; 6, and 401 ten-thousandths. 151.49251.

9. Add 432, and 432 thousandths; 61, and 793 ten-thousandths; 100, and 7794 hundred-thousandths; 6.009; 1000, and 1001 ten-thousandths. 1599.69834.

10. Add 16, and 41 thousandths; 9, and 94 millionths; 33, and 27 hundredths; 8, and 969 thousandths; 32, and 719906 millionths. 100.

11. Add 204, and 9 ten-thousandths; 103, and 9 hundred-millionths; 42, and 9099 millionths; 430, and 99 hundredths; 220.0000009. 999.99999999.

12. Add 35 ten-thousandths; .00035; 35 millionths, and 35 ten-millionths. .0038885.

SUBTRACTION OF DECIMALS

144. Subtraction of decimals is the process of finding the difference between two decimal numbers.

1. From 729.835 subtract 461.5738.

SOLUTION. — Write the numbers so that the two decimal points may be in a column, the units 9 and 1 in the first column to the left, the tenths 8 and 5 in the first column to the right, etc.; then, subtracting as in integers, place the decimal point in the remainder between 8 and 2 under the column of decimal points.

OPERATION

$$\begin{array}{r} 729.835 \\ 461.5738 \\ \hline 268.2612 \end{array}$$

NOTE. — The ten-thousandth place in the minuend may be regarded as occupied by a cipher (§ **140**).

Rule. — 1. *Write the numbers so that the decimal points and figures of the same order may stand in the same column.*

2. *Subtract as in integers.*

3. *Place the decimal point in the remainder under the column of decimal points.*

2.	From 97.5168 subtract 38.25942.	59.25738.
3.	From 20.014 subtract 7.0021.	13.0119.
4.	From 5.03 subtract 2.115.	2.915.
5.	From 24.0042 subtract 13.7013.	10.3029.
6.	From 170.0035 subtract 68.00181.	102.00169.
7.	From .0142 subtract .005.	.0092.
8.	From .05 subtract .0024.	.0476.
9.	From 13.5 subtract 8.037.	5.463.
10.	From 3 subtract .00003.	2.99997.
11.	From 29.0029 subtract 19.003.	9.9999.
12.	From 5 subtract .125.	4.875.
13.	From 1 thousand subtract 1 ten-thousandth.	999.9999.
14.	From 1 subtract 1 millionth.	.999999.
15.	From 25 thousandths take 25 millionths.	.024975.

MULTIPLICATION OF DECIMALS

145. **Multiplication of decimals** is the process of finding the product of numbers involving decimals.

146. Placing the decimal point in the product depends upon the following principle:

Principle. — *The number of decimal orders in the product is equal to the number of decimal orders in both the factors.*

Thus, let the factors be .2 and .03; then, the number of decimal orders in the product will be three. For, $.2 = \frac{2}{10}$ and $.03 = \frac{3}{100}$; the product of .2 by .03 will be the same as the product of $\frac{2}{10}$ by $\frac{3}{100}$. $\frac{2}{10} \times \frac{3}{100} = \frac{6}{1000}$; and $\frac{6}{1000} = .006$. Therefore, $.2 \times .03 = .006$, in which there are three decimal orders.

147. **1.** Multiply 2.149 by 6.34.

SOLUTION.—Multiply as in integers, 2149 by 634. There are three decimal orders in 2.149, and two decimal orders in 6.34; hence, there must be five decimal orders in the product (§ **146**). Therefore, the product is 13.62466.

OPERATION

$$\begin{array}{r} 2.149 \\ \underline{6.34} \\ 8596 \\ 6447 \\ \underline{12894} \\ 13.62466 \end{array}$$

2. Multiply .0276 by .035.

SOLUTION.—Multiply the numerator (§ **134**) 276 by the numerator 35; the result is 9660. There are four decimal orders in .0276, and three decimal orders in .035; hence, there must be seven decimal orders in the product (§ **146**); three ciphers, then, must be prefixed to 9660. Therefore, omitting the cipher on the right (§ **140**) the product is .000966.

OPERATION

$$\begin{array}{r} .0276 \\ \underline{.035} \\ 1380 \\ \underline{828} \\ .0009660 \end{array}$$

3. Multiply 2.075 by 100.

SOLUTION.—Write 2075 and place the decimal point between 7 and 5, two places farther to the right than it is in 2.075.

OPERATION

207.5

NOTE.—To multiply 207.5 by 100, annex a cipher and move the decimal point two places to the right.

Rule.—1. *Find the product of the numerators of the decimals as in integers.*

2. *Point off as many decimal orders in the product as there are decimal orders in both factors.*

NOTES.—1. When the number of figures in the product of the numerators is less than the number of decimal orders required, prefix ciphers (Ex. 2).

2. After placing the decimal point, omit ciphers at the right of the decimal part of the product (Ex. 2).

3. To multiply a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the *right* as there are ciphers in the multiplier. If there are not enough figures annex ciphers.

4. Multiply 33.21 by 4.41.	146.4561.
5. Multiply 32.16 by 22.5.	723.6.
6. Multiply .125 by 9.	1.125.
7. Multiply .35 by 7.	2.45.
8. Multiply .2 by .8.	.16.
9. Multiply .02 by .4.	.008.
10. Multiply .15 by .7.	.105.
11. Multiply 125.015 by .001.	.125015.
12. Multiply .135 by .005.	.000675.
13. Multiply 1.035 by 17.	17.595.
14. Multiply 19 by .125.	2.375.
15. Multiply 4.5 by 4.	18.
16. Multiply .625 by 64.	40.
17. Multiply 61.76 by .0071.	.438496.
18. Multiply 1.325 by .0716.	.09487.
19. Multiply 4.87 by 10.	48.7.
20. Multiply 5.3 by 100.	530.
21. Multiply 17.62 by 100.	1762.
22. Multiply 1.01 by 10.	10.1.
23. Multiply .0001 by 100.	.01.
24. Multiply 1 tenth by 1 hundredth.	.001.
25. Multiply 1 hundred by 1 ten-thousandth.	.01.

26. Multiply 43 thousandths by 21 ten-thousandths. .0000903.

27. Multiply 40000 by 1 millionth. .04.

28. Multiply .09375 by 1.064. .09975.

DIVISION OF DECIMALS

148. Division of decimals is the process of finding the quotient of two numbers involving decimals.

149. Placing the decimal point in the quotient depends upon the following principle:

Principle. — *The number of decimal orders in the quotient is equal to the number of decimal orders in the dividend, less the number in the divisor.*

Thus, let .006 be divided by .03; then the number of decimal orders in the quotient will be one. For $.006 = \frac{6}{1000}$ and $.03 = \frac{3}{100}$; then the quotient of .006 by .03 will be the same as the quotient of $\frac{6}{1000}$ divided by $\frac{3}{100}$. But $\frac{6}{1000} \div \frac{3}{100} = \frac{2}{10}$, and $\frac{2}{10} = .2$. Therefore, $.006 \div .03 = .2$, in which there is one decimal order.

150. **1.** Divide 2.125 by .5.

SOLUTION. — Divide as in integers 2125 by 5. There are three decimal orders in 2.125, and one decimal order in .5; hence there must be two decimal orders in the quotient (§ **149**). Therefore the quotient is 4.25.

OPERATION

$$.5\overline{)2.125}$$
$$4.25$$

2. Divide .048 by .006.

SOLUTION. — Divide the numerator 48 by the numerator 6. There are three decimal orders in .048, and three decimal orders in .006; hence there will be no decimal orders in the quotient (§ **149**). Therefore the quotient is 8.

OPERATION

$$.006\overline{).048}$$
$$8$$

3. Divide .3 by .004.

SOLUTION.—Annex two ciphers to .3; then solve as in Ex. 2.

OPERATION

$$.004\overline{).300}$$
$$75$$

4. Divide 83.1 by 4.

SOLUTION.—Annex two ciphers to the decimal (§ **140**) in order that the division may be performed exactly; then solve as in Ex. 1.

OPERATION

$$4\overline{)83.100}$$
$$20.775$$

5. Divide 2.11 by 3.

SOLUTION.—Annex one or more ciphers to the decimal (§ **140**) in order to carry the division as far as is wanted; then solve as in Ex. 1.

OPERATION

$$3\overline{)2.110}$$
$$.703+$$

6. Divide 475.6 by 100.

SOLUTION.—Write 4756 and place the decimal point between 4 and 7, two places farther to the left than it is in 475.6.

OPERATION

4.756

NOTE.—To divide 4.756 by 100, prefix a cipher; thus, .04756.

Rule.—1. ***Divide the numerator of the dividend by the numerator of the divisor, as in integers.***

2. ***Point off as many decimal orders in the quotient as the number of orders in the dividend exceeds the number in the divisor.***

NOTES.—1. When the number of decimal orders in the dividend is the same as the number in the divisor, the quotient is an integer (Ex. 2).

2. When the number of decimal orders in the dividend is less than the number in the divisor, for convenience in pointing off, make them the same by annexing ciphers to the dividend (Ex. 3).

3. When the division is not exact, it may be continued to any required number of decimal places (Ex. 5).

4. To divide a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the left as there are ciphers in the divisor. If there are not enough figures, prefix ciphers (Ex. 6, Note).

7.	Divide 1.125 by .03.	37.5.
8.	Divide 86.075 by 27.5.	3.13.
9.	Divide 24.73704 by 3.44.	7.191.
10.	Divide 206.166492 by 4.123.	50.004.
11.	Divide .96 by .24.	4.
12.	Divide .0425 by .0085.	5.
13.	Divide 21 by .5.	42.
14.	Divide 2 by .008.	250.
15.	Divide 37.2 by 5.	7.44.
16.	Divide 100.8788 by 454.	.2222.
17.	Divide .000343 by 3.43.	.0001.
18.	Divide 9811.0047 by .108649.	90300.
19.	Divide .21318 by .19.	1.122.
20.	Divide 102048 by .3189.	320000.
21.	Divide .102048 by 3189.	.000032.
22.	Divide 9.9 by .0225.	440.
23.	Divide 872.6 by 100.	8.726.
24.	Divide 4.5 by 1000.	.0045.
25.	Divide 400 by 10000.	.04.
26.	Divide 1 tenth by 10.	.01.
27.	Divide 1 by 1 tenth.	10.
28.	Divide 10 by 1 hundredth.	1000.
29.	Divide 1.7 by 64.	.0265625.
30.	Divide .08 by 80.	.001.
31.	Divide 1.5 by 7.	.2142857 +
32.	Divide 11.1 by 32.76.	.3388278 +
33.	Divide .0123 by 3.21.	.00383177 +

DECIMAL COMPOUND DENOMINATE NUMBERS

151. A decimal is reduced to a lower denomination by multiplication (§ 63, Rule I).

1. Reduce .05 gal. to the decimal of a pint.

OPERATION

$$\begin{array}{r} .05 \\ \underline{4} \\ .20 \\ \underline{2} \\ .4 \end{array}$$

SOLUTION. — To reduce .05 gal. to the decimal of a pint, multiply by 4 and by 2. The result is .4 pint.

2. Reduce .035 pk. to the decimal of a pint. .56 pt.
3. Reduce .0075 bu. to the decimal of a quart. .24 qt.
4. Reduce .005 yd. to the decimal of an inch. .18 in.

152. To find the value of a decimal in integers (§ 125):

1. Find the value of .3125 bu. in integers.

OPERATION

$$\begin{array}{r} .3125 \\ \underline{4} \\ 1.2500 \\ \underline{8} \\ 2.00 \end{array}$$

SOLUTION. — To reduce .3125 bu. to pecks, multiply by 4; the result is 1.25 pk. To reduce .25 pk. to quarts, multiply by 8; the result is 2 qt. Therefore, .3125 bu. equals 1 pk. 2 qt.

2. Find the value of .75 yd. in integers. 2 ft. 3 in.
3. Find the value of .3375 A. in integers. 54 sq. rd.
4. Find the value of .7 lb. Troy in integers. 8 oz. 8 pwt.

153. A decimal is reduced to a higher denomination by division (§ 63, Rule II).

1. Reduce .64 pt. to the decimal of a gallon.

OPERATION

$$\begin{array}{r|l} 2 & .64 \\ 4 & .32 \\ & .08 \end{array}$$

SOLUTION. — To reduce .64 pt. to the decimal of a gallon, divide by 2 and by 4. The result is .08 gal.

2. Reduce .72 qt. to the decimal of a bushel. .0225 bu.

3. Reduce .77 yd. to the decimal of a mile. .0004375 mi.

4. Reduce .25 pt. to the decimal of a gallon. .03125 gal.

Miscellaneous Examples

154. 1. What is the cost of 9 yd. flannel, at \$0.40 per yard, and 12 yd., at \$0.75 per yard? \$12.60.

2. What is the cost of 4.5 yd. of ribbon, at \$0.23 per yard, and 1.5 yd., at \$0.375 per yard? \$1.5975.

3. What is the cost of 16.25 yd. of cloth, at \$2.68 per yard? \$43.55.

4. At \$0.75 per bushel, how much wheat can be bought for \$35.25? 47 bu.

5. At \$2.56 per yard, how much cloth can be bought for \$98.56? 38.5 yd.

6. How much will 13 A. 115 sq. rd. of land cost, at \$17.28 per acre? \$237.06.

7. If 63 gal. of ink cost \$49, how much will 464 gal. cost? \$360.88+.

8. Add .34 yd. 1.07 ft. and 8.92 in. 2 ft. 10 in.

9. Add .625 gal. and .75 qt. 3 qt. .5 pt.

10. From .05 yr. subtract .5 hr. 18 da. 5 hr. 48 min.

11. From .41 da. subtract .16 hr. 9 hr. 40 min. 48 sec.

12. Find the value of .3 yr. in integers. 109 da. 13 hr. 48 min.

13. What is the cost of 343 yd. 2 ft. 3 in. of tubing at \$0.16 per yard? \$55.

METRIC SYSTEM

155. The **metric system** is so called from the *meter*, the unit upon which the system is based. It is a decimal system of weights and measures.

NOTE. — The French originated this system of weights and measures in 1795. Its use has been legalized, in the United States, by act of Congress, and it is in general use by scientific men throughout the world.

The principal units of the metric system are the **meter** for lengths; the **square meter** and **are** for surfaces and land; the **cubic meter** and **stere** for cubic or wood measure; the **liter** for capacity, and the **gram** for weight.

These units are divided and multiplied decimally, the size of the various measures being indicated by the following prefixes: *deca* means 10; *hecto*, 100; *kilo*, 1000; *myria*, 10000; *deci* means .1; *centi*, .01; *milli*, .001. The following table shows that the notation is the same as that of our ordinary system.

Myrias	Kilos	Hectos	Decas	UNITS		decis	centis	millis
2	2	2	2	2	.	2	2	2
Ten-Thousands.	Thousands.	Hundreds.	Tens.	UNITS.		tenths.	hundredths.	thousandths.

Numbers are reduced to lower denominations by removing the decimal point as many places to the right as there are ciphers in the multiplier; and to higher denominations by moving the decimal point similarly to the left.

MEASURE OF LENGTH

156. The **meter** is the unit of length. It is equal to 39.37 inches.

TABLE

10 millimeters, (^{mm})	=	1 centimeter, marked ^{cm}.
10 centimeters	=	1 decimeter, marked ^{dm}.
10 decimeters	=	1 meter, marked ^{m}.
10 meters	=	1 decameter, marked ^{Dm}.
10 decameters	=	1 hectometer, marked ^{Hm}.
10 hectometers	=	1 kilometer, marked ^{Km}.
10 kilometers	=	1 myriameter, marked ^{Mm}.

NOTE.—The measures chiefly used are the **meter** and **kilometer**. The meter, like the yard, is used in measuring cloth and short distances; the kilometer is used in measuring long distances. The meter is nearly one ten-millionth the distance from the equator to the pole.

1. Reduce 5.638^{m} to centimeters.

SOLUTION.—To reduce meters to centimeters, multiply by 100. Write 5638 and place the decimal point between 3 and 8, two orders farther to the right than it is in 5.638. *Ans.* 563.8^{cm}.

2. Reduce 8.753^{Km} to meters.

SOLUTION.—To reduce kilometers to meters, multiply by 1000. *Ans.* 8753^{m}.

3. Reduce 3642.9^{m} to kilometers.

SOLUTION.—To reduce meters to kilometers, divide by 1000. Write 36429 and place the decimal point between 3 and 6, three orders farther to the left than it is in 3642.9. *Ans.* 3.6429^{Km}

4. Reduce 30.75^{m} to centimeters. 3075^{cm}.

5. Reduce 4.5^{Km} to meters. 4500^{m}.

LAND OR SQUARE MEASURE

157. The **are** is the unit of land measure; it is legal at 119.6 square yards.

TABLE

100 centares, $^{(ca)}$ = 1 are, marked ^{a}.
100 ares, = 1 hectare, marked ^{Ha}.

NOTES.—1. An are is 100 **square meters**, marked ^{qm}, that is, a square whose side is 10 meters. The hectare is very nearly 2½ acres.

2. For measuring other surfaces, **squares of the meter** and its subdivisions are used.

1. Reduce 2.625^{Ha} to ares. 262.5^{a}.
2. Reduce 397.8^{a} to hectares. 3.978^{Ha}.
3. Reduce 2500^{ca} to hectares. $.25^{Ha}$.
4. Reduce 3.8^{a} to square meters. 380^{qm}.

MEASURE OF CAPACITY

158. The **liter** is the unit of capacity; it is legal at 1.0567 quarts, liquid measure.

TABLE

100 liters,$(^{l})$ = 1 hectoliter, marked ^{Hl}.

NOTES.—1. The measures commonly used are the **liter** and **hectoliter**. The liter is equal to a cube whose sides are one tenth of a meter. It is very nearly a quart; it is used in measuring milk, wine, etc., in moderate quantities. The hectoliter is about 2 bu. 3¼ pk.; it is used in measuring grain, fruit, roots, etc., in large quantities.

2. The stere, used for measuring wood, is equal to a **cubic meter.**

1. Reduce 2.456^{Hl} to liters. 245.6^{l}.
2. Reduce 873.5^{l} to hectoliters. 8.735^{Hl}.
3. Reduce 1.83^{Hl} to liters. 183^{l}.
4. Reduce 12 steres to cubic meters. $12^{cu.m}$.

MEASURE OF WEIGHT

159. The **gram** is the unit of weight; it is legal at 15.432 grains.

TABLE

1000 **grams**, (^{g}) = 1 kilogram, marked ^{Kg}.
1000 kilograms = 1 metric ton, marked ^{T}.

NOTE.—The weights commonly used are the **gram**, **kilogram**, and **metric ton** or **tonneau**. The gram is the weight of a cube of pure water whose edge is .01 of a meter. It is used in mixing medicines, in weighing the precious metals, and in all cases where great exactness is required. The kilogram—or, as it is commonly called, the "kilo"—is the usual weight for groceries and coarse articles generally; it is very nearly 2⅕ lb. avoirdupois. The metric ton is used for weighing hay and other heavy articles; it is about 204 lb. more than our ton.

1. Reduce 1428.06^g to kilograms. 1.42806^{Kg}.
2. Reduce $.28^{Kg}$ to grams. 280^g.
3. Reduce 1713.5^{Kg} to metric tons. 1.7135^T.
4. Reduce $.00654^{Kg}$ to grams. 6.54^g.

160. TABLE OF VALUES

DENOMINATION	LEGAL VALUE	APPROXIMATE VALUE
Meter.	39.37 inches.	1.1 yards.
Kilometer.	.62137 mile.	⅝ mile.
Square Meter.	1.196 sq. yards.	10¾ sq. feet.
Are.	119.6 sq. yards.	$\frac{1}{40}$ acre.
Hectare.	2.471 acres.	2½ acres.
Cubic Meter.	1.308 cu. yards.	35⅓ cu. feet.
Stere.	.2759 cord.	¼ cord.
Liter.	1.0567 quarts liquid. .908 quart dry.	1 quart.
Hectoliter.	2.8372 bushels.	2 bu. 3⅓ pecks
Gram.	15.432 gr. Troy.	15½ grains.
Kilogram.	2.2046 lb. Av.	2⅕ pounds.
Tonneau.	2204.6 lb. Av.	1 T. 204 lb.

Note. — The legal value is used in solving the following examples:

1. How many yards, feet, etc., are there in 4^m?

OPERATION

$$\begin{array}{r|l} & 39.37 \text{ in.} \\ & \underline{\quad\quad 4} \\ 12 & 157.48 \text{ in.} \\ \hline 3 & 13 \text{ ft. } 1.48 \text{ in.} \\ \hline & 4 \text{ yd. } 1 \text{ ft.} \end{array}$$

Solution. — In 4 meters there are 4 times 39.37 in., which are 157.48 in., 157.48 in. reduced to integers of higher denominations are 4 yd. 1 ft. 1.48 in.

2. What is the value of 36 lb. in kilograms?

Solution. — In 36 pounds there are as many kilograms as 2.2046 lb. are contained times in 36 lb. which are 16.329+.

$36.0000 \div 2.2046 = 16.329+$

3. What is the value of 20^{Km}? 12.4276 mi.

4. How many hectares are there in 160 acres? $64.75+^{Ha}$.

5. What is the value of 49^m? 9 rd. 4 yd. 3.13 in.

6. What is the value of 15^g? 9 pwt. 15.48 gr.

7. How many hectoliters are there in 42 bu.? $14.8+^{Hl}$.

8. How many cords are there in 500 steres? 137.95 C.

9. How many square yards are there in a roll of paper 9^m long and $.5^m$ wide? 5.382 sq. yd.

10. 32^l are how many gallons? 8.4536 gal.

11. What will be the cost of 45^{Ha} of land, at \$3.32 an are? \$14940.

12. A merchant paid \$457.92 for cloth, at \$3 a meter. How many meters did he buy? 152.64^m.

13. A block of marble $.72^m$ long, $.48^m$ wide, and $.5^m$ thick cost \$.864. What is the cost of the marble per cubic meter? \$5.

14. Which is cheaper and how much, to buy cloth at \$2 per meter or at \$1.80 per yard?

$3\frac{3}{20}$¢ per meter cheaper at \$1.80 per yard.

15. How many hectoliters of oats are there in 4685 sacks, each containing 1.6^{Hl}? 7496^{Hl}.

16. If a bushel of oats weighs 32 lb., how many kilograms will a hectoliter weigh, counting 2.8372 bushels to the hectoliter? 41.18^{Kg}.

17. I bought 346.75^{Kg} of coffee for \$194.18. How much did I pay per kilogram? \$0.56.

18. The nickel 5-cent coin weighs 5^{g} and is 2^{cm} in diameter. What would be the weight of enough of these coins laid in a row to make a meter in length? 250^{g}.

19. How much lining 1.85^{m} wide will it take for a garment made of 6.5^{m} of cloth 1.25^{m} wide? $4.39+^{m}$.

20. How many kilometers is it from Cincinnati to Dayton, the distance being 60 miles? $96.56+^{Km}$.

21. A map is 29^{mm} long and 22.4^{mm} wide. What space does it cover? 649.6^{qmm}.

22. The distance between two towns is 13.24037^{Km}. How many steps of $.715^{m}$ each must I take to walk that distance? 18518 steps.

23. How high must a pile of wood be to contain 30 steres if it is 5^{m} long and 2^{m} wide? 3^{m}.

PRACTICAL MEASUREMENTS

161. The following special rules for various measurements are those most commonly used in actual practice.

GOVERNMENT LANDS

A. The unit of land measure is the **acre.**

Government lands are usually divided into rectangular tracts called **townships,** which contain 23040 acres.

Townships are divided into sections, half-sections, quarter-sections, half-quarter-sections, quarter-quarter-sections, and lots.

DIAGRAM No. 1

A TOWNSHIP

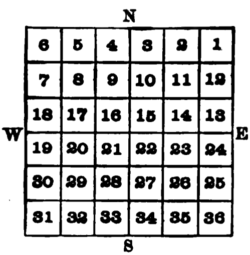


DIAGRAM No. 2

A SECTION

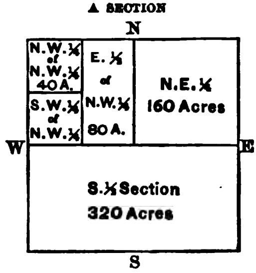


Diagram No. 1 shows the subdivision of a *township* into sections, and how they are *numbered*, commencing at the N.E. corner.

Diagram No. 2 shows the subdivision of a *section*, on an enlarged scale, and how they are *named*.

TABLE

6 mi. × 6 mi. =	36 sq. mi. =	23040	acres = 1	township.
1 mi. × 1 mi. =	1 sq. mi. =	640	acres = 1	section.
1 mi. × ½ mi. =	½ sq. mi. =	320	acres = 1	half-section.
½ mi. × ½ mi. =	¼ sq. mi. =	160	acres = 1	quarter-section.
½ mi. × ¼ mi. =	⅛ sq. mi. =	80	acres = 1	half-quarter-section.
¼ mi. × ¼ mi. =	$\frac{1}{16}$ sq. mi. =	40	acres = 1	quarter-quarter-section.

1. What part of a section is the N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$? How many acres? $\frac{1}{16}$ section, 40 acres.

2. How many acres are there in the W. $\frac{1}{2}$ of N.W. $\frac{1}{4}$? 80 A.

3. If a township of land is equally divided among 360 families, how many acres will each receive? 64 A.

4. If I buy the N. $\frac{1}{2}$ of a section of land at \$3 an acre and sell the W. $\frac{1}{2}$ of my purchase at \$4 an acre and the E. $\frac{1}{2}$ at \$5 an acre, how much do I gain? \$480.

5. How many acres are there in the E. $\frac{1}{2}$ of the N.W. $\frac{1}{4}$ of a section of land and the S. $\frac{1}{2}$ section? 400 A.

CARPETING

B. Carpets are usually either 1 yd. wide or $\frac{3}{4}$ yd. wide.

As there is usually waste in matching the pattern of a carpet and in turning under parts, we cannot, as a rule, find the exact amount of carpet needed by calculating the square yards in the floor; we must find the number of strips of carpet required.

1. How many yards of carpet will be required to carpet a room 20 ft. long by $13\frac{1}{2}$ ft. wide, with carpet $\frac{3}{4}$ yd. wide, if the strips run lengthwise? How many if the strips run across the room?

Solution. — Since the carpet is 27 in. wide and the room $13\frac{1}{2}$ ft. or 162 in. wide, when laid lengthwise there will be needed 6 breadths or strips of carpet of the required length. And since the room is 20 ft. or 240 in. long, it will require as many yards for each

OPERATION

$13\frac{1}{2}$ ft. = 162 in. 20 ft. = 240 in.
$162 \div 27 = 6$ strips.
$240 \div 36 = 6\frac{2}{3}$ yd., length of strips.
$6 \times 6\frac{2}{3}$ yd. = 40 yd. lengthwise, *Ans.*
$240 \div 27 = 8 + = 9$ strips.
$162 \div 36 = 4\frac{1}{2}$ yd., length of strips.
$9 \times 4\frac{1}{2}$ yd. = $40\frac{1}{2}$ yd., crosswise, *Ans.*

strip as 36 in., or 1 yd., is contained times in 240 in., which is $6\frac{2}{3}$ yd. Hence for 6 strips, it will require $6 \times 6\frac{2}{3}$ yd., or 40 yd.

If laid crosswise, as many strips will be needed as 27 in., the width of the carpet, is contained times in 240 in., the length of the room, which is $8\frac{8}{9}$; therefore 9 strips will be required, and $\frac{1}{9}$ will be turned under. As many yards will be needed for each strip as 36 in., or 1 yd., is contained times in 162 in., that is, $4\frac{1}{2}$ yd. Hence for 9 strips there will be required 9 times $4\frac{1}{2}$ yd., or $40\frac{1}{2}$ yd.

Rule. — *Multiply the number of strips required by the length of one strip.*

2. How many yards of carpet 1 yd. wide will be required for a room 12 feet square if the strips run lengthwise? 16 yd.

3. How many yards of carpet $\frac{3}{4}$ yd. wide will cover a floor 17 ft. by 15 ft. if the strips run crosswise? 40 yd.

4. A room 36 ft. long and 18 ft. wide is covered with carpet 1 yd. wide at \$1 per yard. Find the cost. \$72.

5. How many yards of carpet $\frac{3}{4}$ yd. wide will be needed for a room 18 ft. long and 16 ft. wide if the strips run lengthwise, and there is $\frac{1}{5}$ yd. waste in each strip in matching? $49\frac{3}{5}$ yd.

PAPERING ROOMS

C. Wall paper is usually 18 in. wide, and is sold in single rolls 8 yd. long or in double rolls 16 yd. long.

Note. — Part of a roll is always considered as a whole roll.

1. How much would it cost to paper the walls of a room 14 feet long, 7 feet wide, and 8 feet high from baseboard to ceiling, with paper 8 yards long, and $\frac{1}{2}$ a yard wide, costing \$.30 a roll?

Solution. — The distance around the room is 2×14 ft. $+ 2 \times 7$ ft. $= 42$ ft. $= 14$ yd. $= 28$ half-yards, the number of strips required. Since the height is 8 ft., and there are 24 ft. in a roll, 1 roll will make 3 strips; and it will take as many rolls to make 28 strips as 3 is contained times in 28, which is 9+. Therefore, 10 rolls will be required. At \$.30 a roll, they will cost \$3.00.

OPERATION

2×14 ft. $= 28$ ft.
$2 \times \ \ 7$ ft. $= \underline{14}$ ft.
42 ft.

$42 \div 3 = 14$ yd. =
28 half-yards, or strips.
$24 \div 8 = 3$ strips to a roll.
$28 \div 3 = 9+$, or 10 rolls.
$10 \times$ \$.30 = \$3.00, *Ans.*

Rule. — 1. *Find the entire distance around the room in yards. Multiply this by 2 to find the number of half-yards, or strips, since the paper is only half a yard wide.*

2. *Divide the number of half-yards by the number of strips that can be cut from a roll, and the result will be the number of rolls required.*

Note. — Since there are 24 ft. in a roll 8 yd. long, if the distance from baseboard to ceiling is more than 6 ft. and not more than 8 ft., 3 strips can be cut from a roll; if more than 8 ft. and not more than 12 ft., 2; etc. In the former case the divisor would be 3; in the latter, 2.

2. How many rolls of paper 8 yd. long and 18 in. wide will be needed for the walls of a room 30 ft. long, 30 ft. wide, and 12 ft. from baseboard to ceiling? 40 rolls.

3. What will be the cost, at 40¢ a roll, of papering the walls of a room 18 ft. long, 14 ft. wide, and 10 ft. high, with paper $\frac{1}{2}$ yd. wide, 8 yd. long, the baseboard being 9 in. high? \$8.80.

4. How many rolls of paper 8 yd. long and 18 in. wide will be required to paper the walls of a room 18 ft. long, 15 ft. wide, $8\frac{3}{4}$ ft. high, the baseboard being 9 in. high? 15 rolls.

PLASTERING, PAINTING, KALSOMINING, PAVING, ETC.

D. Plastering, painting, kalsomining, and paving are measured by the **square yard.** Glazing is measured by the **square foot** or by the **pane**; and flooring and roofing, by the **square yard** or by the **square of 100 feet.**

1. How much would it cost to plaster the walls and ceilings of a room 16 ft. long, 15 ft. wide, and 11 ft. high, in which there are two doors, 2 ft. 8 in. by 7 ft., and two windows, 2 ft. 10 in. by 6 ft. 2 in., at \$.37 a square yard, deducting half the area of doors and windows?

Solution. — Two of the walls are 16 ft. × 11 ft., two, 15 ft. × 11 ft., and the ceiling, 16 ft. × 15 ft. Adding these, we find the total area to be 922 sq. ft. The area of the two doors is twice 7 ft. × $2\frac{2}{3}$ ft., and of the two windows twice $2\frac{5}{6}$ ft. × $6\frac{1}{6}$ ft. Adding these, the area of doors and windows is 72.2 sq. ft. Deduct half this area, 36.1 sq. ft., from 922 sq. ft.; divide the remainder, 885.9 sq. ft., by 9, to reduce it to square yards; multiply \$.37 by the result, 98.4, and the answer is \$36.41.

OPERATION

$2 \times 16 \text{ ft.} \times 11 \text{ ft.} = 352 \text{ sq. ft.}$
$2 \times 15 \text{ ft.} \times 11 \text{ ft.} = 330 \text{ sq. ft.}$
$16 \text{ ft.} \times 15 \text{ ft.} = \underline{240} \text{ sq. ft.}$
Total area, $= 922$ sq. ft.
$2 \times 2\frac{2}{3} \text{ ft.} \times 7 \text{ ft.} = 37.3 \text{ sq. ft.}$
$2 \times 2\frac{5}{6} \text{ ft.} \times 6\frac{1}{6} \text{ ft.} = \underline{34.9} \text{ sq. ft.}$
$72.2 \text{ sq. ft.} \div 2 =$
36.1 sq. ft., area to be deducted.
$922 \text{ sq. ft.} - 36.1 \text{ sq. ft.} =$
$885.9 \text{ sq. ft.} \quad 885.9 \div 9 = 98.4 \text{ sq. yd.}$
$98.4 \times \$.37 = \$36.41$, *Ans.*

2. A room is 20 ft. 6 in. long, 16 ft. 3 in. wide, 10 ft. 1 in. high. How many yards of plastering are there in it, deducting a fireplace 6 ft. 3 in. by 4 ft. 2 in., a door 7 ft. by 4 ft. 2 in., and two windows, each 6 ft. by 3 ft. 3 in.?
108 sq. yd. 8 sq. ft. 6 sq. in.

3. At 10¢ a square yard, how much will it cost to paint both sides of a partition 15 ft. 6 in. long, 12 ft. 6 in. high?
\$4.31.

4. How much will it cost to pave a rectangular court 21 yd. long and 15 yd. wide, in which a foot path, 5 ft. wide, runs the whole length, the path being paved with flags, at 36¢ per square yard, and the rest with bricks, at 24¢ per square yard? $79.80.

5. A house has three tiers of windows, seven in a tier. The height of the first tier is 6 ft. 11 in.; of the second, 5 ft. 4 in.; of the third, 4 ft. 3 in.; each window is 3 ft. 6 in. wide. How much will the glazing cost, at 16¢ per square foot? $64.68.

6. At 7¢ a square yard for each coat, what is the cost of painting with two coats of paint the walls and ceiling of a room 18 ft. long, 14 ft. wide, and 9 ft. high? $12.88.

7. A floor is 36 ft. 3 in. long, 16 ft. 6 in. wide. How much will it cost to lay it, at $3 a square? $17.97.

ROOFING

E. Shingling and slating are usually estimated by the **square** (100 ft.). Shingles average 4 in. in width, and are usually laid 4 in., $4\frac{1}{2}$ in., or 5 in., to the weather. They are put up in bundles of 250 each. Tinning is estimated by the **square foot**.

1. If shingles are laid 4 in. to the weather, how many square feet will 6 bundles cover? $166\frac{2}{3}$ sq. ft.

2. If shingles are laid $4\frac{1}{2}$ in. to the weather, how many square feet will 10 bundles cover? $312\frac{1}{2}$ sq. ft.

3. Find the cost of tinning the roof of a porch 35 feet long and 8 ft. wide, at 7¢ a square foot. $19.60.

4. Find the cost of slating a roof 50 ft. long, each side of which is 20 ft. wide, at $12.75 per square. $255.

5. At $3.50 per square, find the cost of a roof 40 ft. long, the rafters on each side being 18 ft. 6 in. long? $51.80.

MASONS' AND BRICKLAYERS' WORK

F. Stone masonry is usually measured by the *perch*, $16\frac{1}{2}$ ft. long, $1\frac{1}{2}$ ft. wide, and 1 ft. thick, or $24\frac{3}{4}$ cu. ft. (§ **70**). Bricklaying is measured by the 1000 bricks.

NOTE. — In some places masonry is reckoned by the cubic foot.

1. What is the cost of a stone wall 53 ft. 6 in. long, 12 ft. 6 in. high, 2 ft. thick, at \$2.25 a perch? \$121.59.

2. How many bricks are there in a wall 48 ft. 4 in. long, 16 ft. 6 in. high, 1 ft. 6 in. thick, allowing 20 bricks to the cubic foot? 23925.

3. How many bricks, each 8 in. long, 4 in. wide, 2.25 in. thick, will be required for a wall 120 ft. long, 8 ft. high, and 1 ft. 6 in. thick? 34560.

4. Find the cost of building a wall 240 ft. long, 6 ft. high, 3 ft. thick, at \$3.25 per 1000, each brick being 9 in. long, 4 in. wide, and 2 in. thick. \$336.96.

LUMBER

BOARD MEASURE

G. Board measure is used in measuring all lumber which is sawed into boards, planks, etc.

A **foot,** board measure, is 1 foot long, 1 foot wide, and 1 inch thick. Hence 12 board feet make 1 cubic foot.

Therefore, to find the number of feet in a board, we have the following rule:

Rule. — 1. *Find the surface of the board in square feet.* 2. *Multiply the surface by the thickness in inches.*

1. How many board feet are there in a three-inch board 16 ft. long and 1 ft. 3 in. wide?

SOLUTION. — $16 \times 1\frac{1}{4} \times 3 = 60$. **60 board feet.**

2. How many board feet are there in an inch board 16 ft. long and 15 in. wide. 20 ft.

3. How many board feet are there in a two-inch plank 12 ft. 6 in. long and 2 ft. 3 in. wide? $56\frac{1}{4}$ ft.

4. How many board feet are there in a piece of scantling 15 ft. long, 4 in. wide, and 3 in. thick? 15 ft.

5. How many feet of inch boards will a stick of timber 12 ft. long and 2 ft. square make? 576 ft.

6. How many board feet are there in 2 joists 17 ft. long, 11 in. wide, and 3 in. thick? $93\frac{1}{2}$ ft.

7. How many board feet are there in a three-inch board, 18 ft. long, and 8 in. wide at one end, and 10 in. at the other?

SOLUTION. — (8 in. + 10 in.) $\div$ 2 = 9 in. or $\frac{3}{4}$ ft. average width. $18 \times \frac{3}{4} \times 3 = \frac{162}{4} = 40\frac{1}{2}$ board feet. *Ans.*

8. How many board feet are there in an inch board, 12 ft. 6 in. long, 1 ft. 3 in. wide at one end, and 11 in. wide at the other end? $13\frac{13}{24}$ ft.

9. How many board feet are there in a stick of timber 36 ft. long, 10 in. thick, 12 in. wide at one end, and 9 in. wide at the other? 315 ft.

MEASUREMENT OF LOGS

H. There are as many board feet in a **log** as in the largest piece of timber that can be sawed from it. A log

12 feet

4 board feet

1 ft. long therefore contains $\frac{1}{12}$ as many board feet as there are square inches on its smaller squared end.

NOTE. — When the log is sawed into boards $\frac{1}{5}$ must be deducted for waste.

1. Find the number of feet of lumber in a log 8 ft. long, if the smaller squared end contains 24 square inches.

OPERATION

$\frac{1}{12} \times 24 \times 8 = 16$ feet of lumber. *Ans.*

2. How many board feet can be sawed from a log 18 ft. long, if the smaller squared end contains 50 square inches?

OPERATION

$\frac{1}{12} \times 50 \times 18 = 75$ feet; $75 - (\frac{1}{5}$ of $75) = 60$ board feet. *Ans.*

3. How many feet of lumber are there in a board 12 ft. long, if the smaller squared end contains 4 square inches? 4 feet.

4. How many board feet can be sawed from a log 20 ft. long, whose smaller squared end contains 18 square inches? 24 board feet.

5. How many board feet can be sawed from a log 40 ft. long, whose smaller squared end contains 144 square inches? 384 board feet.

Wood is usually cut for marketing into 4 feet sticks. Two lengths of such wood piled 4 ft. high and 4 ft. wide make a **cord** or 128 cu. ft. ($2 \times 4 \times 4 \times 4 = 128$). 16 cu. ft. or $\frac{1}{8}$ of a cord is a **cord foot.**

6. At $4 per cord what is the cost of a load of wood measuring 4 ft. by 4 ft. by 2 ft.? $1.

7. Find the cost at $3.50 per cord of a load of wood measuring 12 ft. by 6 ft. by 4 ft. $7.87½.

Laths are 4 ft. long, 1½ in. wide, and are laid ⅜ in. apart at the sides, and close together at the ends. They are put up in bundles of 50 each. **Clapboards** are usually cut 6 in. wide, and are put up in bundles of 25 each.

8. How many bundles of laths will be required for a ceiling 16 ft. by 15 ft.? 8 bundles.

9. How many square yards will be laid by two bundles of 50 laths each? $6\frac{17}{18}$ sq. yd.

10. How many clapboards 12 ft. long laid 5 in. to the weather will be required to cover 140 sq. ft.?

28 clapboards.

CAPACITY OF BINS, CISTERNS, ETC.

I. There are 2150.4 cubic inches in a **bushel** (§ 61) and 231 cubic inches in a **gallon** (§ 64).

To find the number of bushels:

Rule. — *Divide the volume in cubic inches by* 2150.4.

To find the number of gallons:

Rule. — *Divide the volume in cubic inches by* 231.

1. How many bushels are there in a bin 15 ft. long, 5 ft. wide, and 4 ft. deep? 241 + bu.

2. How many gallons are there in a trough 10 ft. long, 5 ft. wide, and 4 ft. deep? 1496 + gal.

3. How high must a bin 5 feet square be made to contain 100 bushels? 4.98 ft.

4. How many bushels can be put in a bin 11 ft. long, 4 ft. wide, and 5 ft. deep? 176.78 bu.

5. How many gallons will a tank hold that is 4 ft. by 5 ft., and 6 ft. deep? 897.66 gal.

NOTE. — A bushel equals nearly $1\frac{1}{4}$ cu. ft., therefore, $\frac{4}{5}$ the number of cubic feet will give the approximate number of bushels. $7\frac{1}{2}$ gal. are allowed in practice for 1 cu. ft., therefore, $7\frac{1}{2}$ times the number of cubic feet will give the approximate number of gallons.

6. Find approximately how many bushels can be put into a bin 10 ft. long, 5 ft. wide, and 5 ft. deep. 200 bu.

7. Find approximately the number of gallons a tank will hold that is 14 ft. long, 4 ft. wide, and 2 ft. deep. 840 gal.

PERCENTAGE

162. Any **per cent** of a number is so many hundredths of it.

Thus, 1 per cent of a number is $\frac{1}{100}$ of it, 2 per cent is $\frac{2}{100}$, etc.

NOTE. — Per cent is from the Latin *per centum*, by the hundred.

The sign of per cent is %, read *per cent.*

Thus, 5% is read *five per cent.*

The following expressions are equivalent:

One	per cent,	1%,	is	$\frac{1}{100}$	or .01
Two	per cent,	2%,	is	$\frac{2}{100} = \frac{1}{50}$,	or .02
Three	per cent,	3%,	is	$\frac{3}{100}$	or .03
Four	per cent,	4%,	is	$\frac{4}{100} = \frac{1}{25}$,	or .04
Five	per cent,	5%,	is	$\frac{5}{100} = \frac{1}{20}$,	or .05
Six	per cent,	6%,	is	$\frac{6}{100} = \frac{3}{50}$,	or .06

NOTE. — Per cent which is expressed as a mixed number, may be reduced to equivalent expressions as follows; $4\frac{1}{2}\% = .045$. Or $4\frac{1}{2}\% = \frac{4\frac{1}{2}}{100} = \frac{9}{2} \div 100 = \frac{9}{200}$.

Express the following as common fractions and as decimals:

1. 10%.	$\frac{1}{10}$ and .10.		**7.** $6\frac{1}{4}\%$.	$\frac{1}{16}$ and .0625.	
2. 15%.	$\frac{3}{20}$ and .15.		**8.** $12\frac{1}{2}\%$.	$\frac{1}{8}$ and .125.	
3. 20%.	$\frac{1}{5}$ and .20.		**9.** $18\frac{3}{4}\%$.	$\frac{3}{16}$ and .1875.	
4. 30%.	$\frac{3}{10}$ and .30.		**10.** $37\frac{1}{2}\%$.	$\frac{3}{8}$ and .375.	
5. 50%.	$\frac{1}{2}$ and .50.		**11.** $56\frac{1}{4}\%$.	$\frac{9}{16}$ and .5625.	
6. $2\frac{1}{2}\%$.	$\frac{1}{40}$ and .025.		**12.** $87\frac{1}{2}\%$.	$\frac{7}{8}$ and .875.	

NOTE. — Common fractions may be reduced to hundredths by § **106**, and then read as per cent. Thus $\frac{1}{6} = .16\frac{2}{3}$ or $16\frac{2}{3}\%$.

How many per cent are equivalent to the following fractions?

1. $\frac{2}{25}$.	8%.	**6.** $\frac{1}{4}$.	25%.
2. $\frac{3}{25}$.	12%.	**7.** $\frac{2}{5}$.	40%.
3. $\frac{4}{25}$.	16%.	**8.** $\frac{3}{4}$.	75%.
4. $\frac{1}{30}$.	$3\frac{1}{3}\%$.	**9.** $\frac{1}{3}$.	$33\frac{1}{3}\%$.
5. $\frac{1}{12}$.	$8\frac{1}{3}\%$.	**10.** $\frac{7}{16}$.	$43\frac{3}{4}\%$.

The per cent equivalents of the following fractions should be committed to memory (see § **122**):

$\frac{1}{2} = 50\%$	$\frac{1}{5} = 20\%$	$\frac{5}{6} = 83\frac{1}{3}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$
$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{2}{5} = 40\%$	$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{1}{9} = 11\frac{1}{9}\%$
$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{3}{5} = 60\%$	$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{1}{11} = 9\frac{1}{11}\%$
$\frac{1}{4} = 25\%$	$\frac{4}{5} = 80\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{1}{12} = 8\frac{1}{3}\%$
$\frac{3}{4} = 75\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{1}{16} = 6\frac{1}{4}\%$

NOTE. — When any of the per cents in the above table appear in a problem, it will usually be found easier to change the per cent to its fractional equivalent, and solve by the rule for fractions which applies to the case.

163. **Percentage** embraces the various operations with per cent.

In percentage three quantities are considered. (1) the *base*, (2) the *rate*, and (3) the *percentage*.

The **base** is the number upon which the per cent is estimated.

The **rate** is the per cent when expressed as a common fraction or as a decimal.

The **percentage** is the result of taking the per cent of the base.

Any two of these quantities being given, the third may be found. There are *four cases*.

CASE I

164. Given the base and the rate, to find the percentage:

1. What is 25% of 32?

OPERATION

SOLUTION. — 25% is $\frac{1}{4}$ (§ 162). $\frac{1}{4}$ of 32 is 8.

$$25\% = \tfrac{1}{4}$$
$$\tfrac{32}{1} \times \tfrac{1}{4} = 8$$

2. What is 7% of 162?

OPERATION

SOLUTION. — 7% is .07 (§ 162). Multiplying 162 by .07, the result is 11.34.

$$\begin{array}{r} 162 \\ .07 \\ \hline 11.34 \end{array}$$

Rule. — *Multiply the base by the rate; the product will be the percentage.*

NOTE. — Whether the rate should be expressed as a common fraction, or as a decimal, must be a matter of judgment. That form of expression is best which is simplest or most convenient in the given example.

What is:

3.	1% of 278?	2.78.	13.	8% of 11?	.88.
4.	2% of 180?	3.6.	14.	$8\frac{1}{3}$% of 384?	32.
5.	3% of 97?	2.91.	15.	10% of 57?	5.7.
6.	$3\frac{1}{3}$% of 165?	5.5.	16.	$12\frac{1}{2}$% of 292?	36.5.
7.	$3\frac{3}{4}$% of 240?	9.	17.	15% of 95?	14.25.
8.	4% of 140?	5.6.	18.	17% of 53.4?	9.078.
9.	5% of 118?	5.9.	19.	$18\frac{3}{4}$% of 11.2?	2.1.
10.	$5\frac{1}{3}$% of 150?	8.	20.	20% of 9.85?	1.97.
11.	6% of 250?	15.	21.	25% of 43?	10.75.
12.	$6\frac{2}{3}$% of 450?	30.	22.	$33\frac{1}{3}$% of 6.93?	2.31.

23.	45% of 5.7?	2.565.	27.	$\frac{7}{16}$% of 144?	.63.
24.	50% of 38.75?	19.375.	28.	125% of 36?	45.
25.	$\frac{1}{2}$% of 456?	2.28.	29.	208% of 650?	1352.
26.	$\frac{3}{8}$% of 464?	1.74.	30.	450% of 12?	54.

CASE II

165. Given the base and the percentage, to find the rate:

1. What per cent of 8 is 2?

OPERATION

$\frac{2}{8} = \frac{1}{4}$
$\frac{1}{4} = 25\%$

SOLUTION.—2 is $\frac{1}{4}$ of 8 (§ **120**). $\frac{1}{4}$ is 25%.

2. What per cent of 56 is 3.5?

OPERATION

$3.5 \div 56 = .0625$
$.0625 = 6\frac{1}{4}\%$

SOLUTION.—Dividing 3.5 by 56 (§ **120**), the result is .0625. .0625 is $6\frac{1}{4}$%.

EXPLANATION.—One per cent of 56 is .56; 3.5 is as many per cent as .56 is contained times in 3.5.

Rule.—*Divide the percentage by the base; the quotient will be the rate.*

What per cent of:

3.	15 is 3?	20%.	11.	243 is 8.505?	$3\frac{1}{2}$%.
4.	50 is 6?	12%.	12.	2 is .002?	$\frac{1}{10}$%.
5.	75 is 4.5?	6%.	13.	3532 is 13.245?	$\frac{3}{8}$%.
6.	9 is 3?	$33\frac{1}{3}$%.	14.	$\frac{4}{5}$ is $\frac{3}{25}$?	15%.
7.	25 is .25?	1%.	15.	$\frac{2}{3}$ is $\frac{2}{15}$?	20%.
8.	142.6 is 7.13?	5%.	16.	$\frac{16}{21}$ is $\frac{2}{7}$?	$37\frac{1}{2}$%.
9.	9 is 9?	100%.	17.	$11\frac{2}{3}$ is $5\frac{1}{4}$?	45%.
10.	9 is 13.5?	150%.	18.	$57\frac{7}{9}$ is $10\frac{5}{6}$?	$18\frac{3}{4}$%.

CASE III

166. Given the rate and the percentage, to find the base:

1. 15 is 25% of what number?

SOLUTION. — 25% is $\frac{1}{4}$ (§ **162**). Since 15 is $\frac{1}{4}$ of some number, the number is $4 \times 15 = 60$. (See § **121**.)

OPERATION

$25\% = \frac{1}{4}$
$15 \times 4 = 60$

2. 4.93 is 17% of what number?

SOLUTION. — 17% is .17 (§ **162**). Since some number multiplied by .17 gives the product 4.93, the number is 4.93 divided by .17 or 29. (See § **121**.)

OPERATION

$17\% = .17$
$4.93 \div .17 = 29$

Rule. — *Divide the percentage by the rate; the quotient will be the base.*

3.	60	is 20% of what number?	300.
4.	90	is 75% of what number?	120.
5.	85	is 125% of what number?	68.
6.	7.13	is 23% of what number?	31.
7.	20.23	is 34% of what number?	59.5.
8.	23.5	is 47% of what number?	50.
9.	45	is $1\frac{1}{2}\%$ of what number?	3000.
10.	2.25	is $12\frac{1}{2}\%$ of what number?	18.
11.	$\frac{3}{4}$	is 250% of what number?	$\frac{3}{10}$.
12.	$14\frac{2}{7}$	is $16\frac{2}{3}\%$ of what number?	$85\frac{5}{7}$.

CASE IV

167 Given the rate and the sum or the difference of the base and percentage, to find the base:

1. A number, plus 35% of itself, equals 675. What is the number?

SOLUTION. — 35% is .35. The number plus .35 of itself equals 1.35 of it; then, 1.35 of the number is 675, and the number itself is 675 divided by 1.35, or 500.

OPERATION

$35\% = .35$
$1 + .35 = 1.35$
$675 \div 1.35 = 500$

2. A number, minus 5% of itself, equals 57. What is the number?

OPERATION

$$5\% = \tfrac{1}{20}$$
$$\tfrac{20}{20} - \tfrac{1}{20} = \tfrac{19}{20}$$
$$\frac{\overset{3}{\cancel{57}}}{1} \times \frac{20}{\cancel{19}} = 60$$

SOLUTION. — 5% is $\frac{1}{20}$. The number minus $\frac{1}{20}$ of itself equals $\frac{19}{20}$ of it; then, $\frac{19}{20}$ of the number is 57, $\frac{1}{20}$ of it is 3, and the number is $20 \times 3 = 60$.

Rule. — *Divide the sum by* 1 *plus the rate, or divide the difference by* 1 *minus the rate; the quotient will be the base.*

3. 721 is 3% greater than a certain number. What is the number? 700.

4. 68 is 66% less than what number? 200.

5. What number, increased by 25% of itself, amounts to 2125? 1700.

6. What number, diminished by 6% of itself, is equal to 7.52? 8.

7. 8250 is $37\frac{1}{2}\%$ greater than what number? 6000.

8. What fraction, less 10% of itself, equals $\frac{3}{8}$? $\frac{5}{12}$.

9. 6.6 is 20% more than what number? 5.5.

168. Formulas for the four cases of Percentage:

Let b represent the base, r the rate, and p the percentage. Then,

Case I. $b \times r = p$.

Case II. $p \div b = r$.

Case III. $p \div r = b$.

Case IV. $\dfrac{b+p}{1+r} = b$. $\dfrac{b-p}{1-r} = b$.

Miscellaneous Examples

169. **1.** I had \$800 in bank and drew out 36% of it. How much had I left? \$512.

2. A man had \$300. After he had spent \$225, what per cent had he left? 25%.

3. A merchant withdrew 40% of his deposits, leaving \$3000 in the bank. What amount did he withdraw? \$2000.

4. A grain dealer sold corn for 56¢ a bushel, which was 40 per cent more than it cost him. What was the cost per bushel? 40¢.

5. A man sold a horse for \$175, which was $12\frac{1}{2}$% less than the horse cost. How much did the horse cost? \$200.

6. A grocer bought 4 sacks of coffee of 75 pounds each; $12\frac{1}{2}$% was lost by waste. How much was the remainder worth at 35 cents per pound? \$$91.87\frac{1}{2}$.

7. A man owed \$500; he paid \$425. What per cent of the debt remained unpaid? 15%.

8. A speculator invested 75% of his estate in bonds, and the remainder of it, amounting to \$5000, in real estate. How much did he invest in bonds? \$15000.

9. A farmer owned a farm of 250 A. 86 sq. rd., which was $12\frac{1}{2}$% more than his neighbor owned. How much land did his neighbor own? 222 A. 112 sq. rd.

10. A flock of 160 sheep increased 35% in one year. How many were then in the flock? 216.

11. A miller takes for toll 6 qt. from every 5 bu. of wheat ground. What per cent does he take? $3\frac{3}{4}$%.

12. A farmer owning 45% of a tract of land sold 540 acres, which was 60% of what he owned. How many acres were there in the tract? 2000 A.

13. If a man gained 7% by selling some goods for \$371.29, what was the cost of the goods? \$347.

14. A's salary is \$800 a year; he spends 18% of it for rent, 15% for clothing, 23% for provisions, and 12% for sundries. How much does he save annually? \$256.

15. A pupil at an examination answered 17 of the 20 questions correctly. What per cent did he get? 85%.

16. 2 bu. 3 pk. are $33\frac{1}{3}\%$ of what quantity? 8 bu. 1 pk.

17. The number of pupils attending school on a certain day was 37; this was $7\frac{1}{2}\%$ less than the number enrolled. How many were enrolled? 40.

18. The five-cent piece weighs 5 grams, of which 1.25 g are nickel and the remainder copper. What is the per cent of copper? 75%.

19. A man sold a horse for \$150, which was 25% more than it cost him. If he had sold the horse for \$200, how many per cent would it have been more than it cost him? $66\frac{2}{3}\%$.

20. Mr. Crane, who had \$10000 in bank, drew out first 20%, then 25% of the remainder, and then deposited 5% of what he had drawn out altogether. How much had he then in the bank? \$6200.

21. At 6 A.M. a man started on a trip of 280 miles in a train running 35 miles an hour. What per cent of the distance had he covered by 9 A.M.? $37\frac{1}{2}\%$.

22. An automobile was sold for \$2560, which was 25% more than the cost. Find the cost. \$2048.

23. The product of two numbers is 12000, and the first factor is 200. What per cent of the first is the second? 30%.

24. If an ordinary telegraphic dispatch to a newspaper costs \$49.52, and the same message costs only $87\frac{1}{2}\%$ as much by wireless telegraphy, what is the cost of the latter transmission? \$43.33.

25. In a spelling match John spelled correctly 265 out of 275 words. What per cent of the words did he misspell? What per cent did he spell correctly?

$3+\%$; $96+\%$.

26. A piece of cloth is 48 yards long. If it shrinks $6\frac{1}{4}\%$ in sponging, what will be its length after it is sponged? 45 yd.

27. By the eruption of Mt. Pelée in the island of Martinique, a man who owned property in St. Pierre lost \$16500, which was $16\frac{2}{3}\%$ of his entire fortune. Find the amount of his fortune before and after the loss.

\$99000 before loss, \$82500 after loss.

28. What number increased by $14\frac{2}{7}\%$ of itself equals 16088? 14077.

29. What number decreased by $14\frac{2}{7}\%$ of itself equals 3648? 4256.

30. After spending $12\frac{1}{2}\%$ of the money in his purse a man found that he had \$1407 left. How much had he at first? \$1608.

31. In 1902 the population of a city was 14280, which was $16\frac{2}{3}\%$ more than the population in 1892. What was the population in 1892? 12240.

APPLICATIONS OF PERCENTAGE

170. The applications of percentage may be divided into *two classes:* (1) Those *without* the element of time; (2) those *with* the element of time.

The most important applications of the first class are *Mercantile Transactions* and *Stock Transactions.*

The most important applications of the second class are *Interest* and *Discount.*

Percentage enters to a greater or less extent into the calculations of *Exchange*, *Insurance*, *Taxes*, etc.

The principles of percentage apply directly to applications of the first class in accordance with the following general rule:

Rule. — 1. *Ascertain the quantities which correspond to base, percentage, and their sum or their difference.*

2. *Note the quantities given and the quantities required.*

3. *Apply the proper case of percentage to the given example.*

MERCANTILE TRANSACTIONS

171. Mercantile transactions relate to the purchase and sale of merchandise.

Price is the value of anything in money.

Merchandise is bought and sold at *wholesale* and at *retail* prices.

The **wholesale** price is the price of merchandise in large quantities.

The **retail** price is the price of merchandise in small quantities.

Note. — Wholesale merchants buy and sell merchandise at wholesale prices. Retail dealers distribute merchandise of every description to the users or consumers of it at retail prices.

The chief mercantile transactions involving an application of percentage are (1) *Commission*, (2) *Trade Discount*, and (3) *Profit and Loss.*

NOTE. — Wholesale merchants buy and sell merchandise largely through agents, who receive salaries or a commission for their services. Buyers at wholesale are sometimes allowed discounts upon their purchases. Merchants usually make a profit or suffer a loss in their transactions.

COMMISSION

172. An **agent** is a person intrusted with the business of another.

NOTE. — The person who employs the agent, in reference to him, is called the principal.

A **commission merchant** buys and sells merchandise for another.

NOTE. — The person to whom merchandise is sent to be sold is termed the **consignee**; the person who sends it is termed the **consignor**; while the merchandise itself is called a **consignment**.

The **commission** is the sum paid an agent for transacting business.

The **charges** are expenses incurred by an agent in transacting business.

The **net proceeds** is the sum remaining after deducting the commission and charges.

The value of the materials in the business transacted is the *base;* the commission is the *percentage;* and the net proceeds is the *base less the percentage.*

1. An agent whose commission is 5% receives how much upon a sale of goods amounting to $240? $12.

2. An auctioneer received $11.50 for selling a lot of goods amounting to $460. What per cent commission did he receive? $2\frac{1}{2}$%.

3. At a commission of $2\frac{1}{2}$% a commission merchant received 8.12\frac{1}{2}$ for selling 25 barrels of molasses. For how much per barrel did he sell the molasses? $13.

4. An agent sent his employer a bill for $210, which included the goods he purchased and his commission of 5%. What sum did he expend for goods? $200.

5. What are the net proceeds on a sale of goods amounting to $180, at 4% commission? $172.80.

6. A lawyer received $11.25 for collecting a debt. His commission being 5%, what was the amount of the debt? $225.

7. An agent receives $1323.54 in settlement of a bill for goods purchased and commission at 8%. What is his commission? $98.04.

8. A commission merchant whose commission is 3% sells 250 bbl. pork, at $15 per barrel; 350 bbl. flour, at $3.50 per barrel; and 100 bbl. apples, at $3.64 per barrel. What sum does he remit the owner? $5178.83.

TRADE DISCOUNT

173. Merchandise may be sold at a *net price* or *at a discount* from published *list* or *regular price.*

A **list** or **regular price** is an established price fixed by the seller as a basis upon which to calculate discounts.

A **net price** is a fixed price from which no discount is allowed, or it is the list price less the discounts.

The **discount** is the deduction from the list or regular price.

The discount is expressed as so many per cent *off* or as so many *off.*

Thus, 20% off, or 20 off, means at a discount of 20% from the price.

There may be: 1st. A single discount, as 5% or 5 off. 2d. Two or more successive discounts.

Thus, the expression 20 and 5 % off means, first, a discount of 20 % from the price, and then a discount of 5 % from the remainder. The expression 25, 10, and 5 % off means three successive discounts.

NOTE. — The per cent is sometimes expressed as a common fraction. Thus, $\frac{1}{8}$ off means $12\frac{1}{2}$ % off; $\frac{1}{3}$ and 5 off means $33\frac{1}{3}$ and 5 off.

The price of the seller is the *base;* the sum of all the discounts is the *percentage;* and the price of the buyer or price paid is the *base less the percentage.*

1. A bill of goods amounted to $225.50 list. 20% off being allowed, how much was paid for the goods? $180.40.

2. A bill of articles amounted to $725.16. The purchaser being allowed $\frac{1}{3}$ and 5 off, how much did he pay? $459.27.

3. On a bill of goods amounting to $500, what is the difference between a discount of 35% and three successive discounts of 20%, 10%, and 5%? $17.

4. Find a single discount equivalent to two successive discounts of 20% and 5% on $500. On any sum. 24%.

NOTE. — 20 % + 5 % of 80 % will be the discount required.

5. The list price of an article at three different stores is $1000. One store offers discounts of 20%, 10%, and 5%; the second store 5%, 10%, and 20%; and the third store 10%, 20%, and 5%. Which is the best offer? No difference.

6. I bought 100 doz. bindings at 60¢ per dozen for 40, 10, and $7\frac{1}{2}$% off. How much did I pay for them? $29.97.

7. A retail dealer buys 10 dozen boxes of paper for $50 list, and gets off 50, 10, and 10%, besides an additional 2% for cash. How much did he pay per dozen for the paper? $1.98.

PROFIT AND LOSS

174. The **cost** is the price paid for goods.

The **selling price** is the price received for goods.

NOTE. — The cost to the consumer is the selling price of the merchant, and the cost to the retail dealer is the selling price of the wholesale dealer.

Goods are usually sold *at a profit* or *at a loss.*

The **profit** is what the goods sell for more than they cost.

The **loss** is what the goods sell for less than they cost.

The cost is the *base;* the profit or the loss is the *percentage;* and the selling price is the *sum or the difference of the base and percentage.*

1. A merchant's profit on a piece of cloth which cost \$40 is 10%. For how much does he sell it? \$44.

2. Ribbon that cost 6¢ a yard is sold for 5¢ a yard. What is the per cent of loss? $16\frac{2}{3}$%.

3. A grocer, by retailing coffee at 27 cents per pound, gains $12\frac{1}{2}$%. What is the cost per pound? 24¢.

4. By selling a lot of goods at a loss of 4% the loss on the entire lot was \$15.30. How much did the goods cost? \$382.50.

5. To make a profit of $37\frac{1}{2}$%, at what price must a dry goods merchant sell shawls that cost \$8? \$11.

6. A bookseller sells a grammar for 90¢ which costs 75¢. What is his gain per cent? 20%.

7. What is the cost of tea which, when sold at $6\frac{1}{4}$% profit, yields a profit of 5¢ per pound? 80¢.

8. A grocer sells flour at \$4.75 per barrel, making a profit of $18\frac{3}{4}$%. What is the cost? \$4.

9. I sold silk at \$1.35 per yard, and lost 10%. At what price per yard should I have sold it to make a profit of $16\frac{2}{3}$%? \$1.75.

10. A peddler bought a stock of goods for \$874, and disposed of them at a profit of 25%. How much money did he make? \$218.50.

11. If a bookseller makes 25¢ on a book that he sells for \$1.75, what is his per cent of profit? $16\frac{2}{3}$%.

12. A dealer sold two horses for \$150 each; on one he gained 25%, and on the other he lost 25%. How much did he lose by the transaction? \$20.

13. A merchant reduced the price of a certain piece of cloth 5¢ per yard, and thereby reduced his profit on the cloth from 10% to 8%. What was the cost of the cloth per yard? \$2.50.

14. A speculator bought 10000 bushels of corn, at 60¢ per bushel. In a few days corn advanced in price, and he sold 7000 bushels, at 65¢ per bushel. Corn then fell in price, and he disposed of the remainder at 55¢ per bushel. What per cent profit did he get out of the transaction? $3\frac{1}{3}$%.

15. A speculator in real estate sold a house and lot for \$12000, thereby gaining $33\frac{1}{3}$% on the cost. He then invested the \$12000 in city lots, which he was obliged to sell at a loss of $33\frac{1}{3}$%. How much did he lose by the two transactions? \$1000.

Miscellaneous Examples

175. 1. A bookseller purchases books from the publisher at 20% off the list price. If he retails them at the list price, what will be his per cent of profit? 25%.

2. A grocer bought 5 half chests of tea of 74 lb. each, at 45¢ per pound, at 2% off for cash. If he retails it at $12\frac{1}{8}\%$ advance, what will be his profit? $20.12.

3. I bought 5 cases of men's boots, containing 12 pairs each, for $45 per case, 5% off for cash; I retailed them at $4.25 per pair. What was my profit? $41.25.

4. I sold a case of hats containing 3 dozen, on which I had received a discount of 10%, and made a profit of $12\frac{1}{2}\%$ or $37\frac{1}{2}$¢ on each hat. What was the wholesale merchant's price per case? $120.

5. A merchant buys 100 packages of pins, of 12 papers each, for $1 per package, 60, 5, and 5% off; if he retails them so as to make a profit of $23.90, for how much a paper will he sell them? 5¢.

6. A contractor bought 80 horses for government, at $125 apiece; the agent's commission was such that the horses cost the government $10250. What per cent was the commission? $2\frac{1}{2}\%$.

7. A dealer in notions buys 60 gross shoestrings, at 70¢ per gross, list, for 50, 10, and 5% off. If he sells them at 20, 10, and 5% off list, what will be his profit? $10.77.

8. A man bought 50 gross of buttons for 25, 10, and 5% off, and disposed of the lot for $35.91, at a profit of 12%. What was the list price of the buttons per gross? $1.

STOCKS AND BONDS

176. A **stock company** is an association of individuals under the laws of a state, for the purpose of transacting business.

The **stock** is the capital invested in the business. It is divided into a number of equal transferable parts called shares. A share is usually $100.

NOTES.—1. The capital of banks, of railroad, insurance, telegraph and other companies is held in this way.

2. A share is sometimes $50 or some other number, but stocks are quoted, in the New York market, invariably as $100 to the share. In the examples in this book, unless otherwise specified, all shares are to be considered as $100 shares.

The **stockholders** are the owners of the stock.

A **bond** is a written promise, under seal, to pay a certain sum of money at a specified time.

NOTES.—1. Bonds are the notes of the government and of the various corporate bodies which are allowed to issue them; usually they bear a given rate of interest and are payable within a specified time.

2. They are named by giving the name of the corporation, the rate of interest they bear, and sometimes the date at which they become due. Thus, Baltimore and Ohio $3\frac{1}{2}$'s are bonds of the Baltimore and Ohio Railroad Co. yielding $3\frac{1}{2}\%$ interest. U.S. 4's 1907 are United States bonds bearing 4% interest and due in 1907.

3. Coupon bonds have certificates of interest attached to them, which are torn off and collected as the interest becomes due.

177. The **par value** of stocks and bonds is the value given on their face. The **market value** is the sum for which they sell at a given time.

Stock is **at par** when it sells for its par value; **above par**, or at a premium, when it sells for more than its par value; and **below par**, or at a discount, when it sells for less than its par value.

The market value or selling price of stocks varies from day to day, and is ascertained by consulting the stock quotations in the newspapers. Stocks are quoted in the papers as being at 34, 80, 108, etc.; and this means that a share, the par value of which is $100, can be bought for $34, $80, $108, etc.; or that $1 worth of the stock will cost $.34, $.80, $1.08. In the first two cases the stock is at a discount, and in the third at a premium.

178. A **stock broker** is a person who buys and sells stocks or bonds as the agent of another. His commission, called **brokerage**, is always calculated on the par value of the stock.

NOTE. — Brokerage in New York is usually $\frac{1}{8}$%. Elsewhere it may be as high as $\frac{1}{4}$%.

179. A **dividend** is a sum of money paid to the stockholders, in proportion to their shares, from the profits of the business, and an **assessment** is a sum paid by the stockholders, in proportion to their shares, to meet the necessary expenses or losses of the business.

180. The par value is the *base;* brokerage, premium, discount, dividends, and assessments are each a *percentage;* the market value is the *amount* or *difference.*

1. A broker bought for me 75 shares New York Central stock. What was the brokerage at $\frac{1}{4}$%? $18.75.

2. The brokerage for buying 50 shares of Union Pacific stock was $6.25. What was the per cent? $\frac{1}{8}$%.

3. At $\frac{1}{4}$% brokerage a broker received $10 for making an investment in bank stock. How many shares did he buy? 40.

4. A broker buys 17 shares United States Steel Company stock. What is his brokerage at $\frac{1}{4}$%? $4.25.

5. The brokerage on 95 shares of Metropolitan Street Railway stock is 11.87\frac{1}{2}$. What is the per cent? $\frac{1}{8}$%.

6. A broker received $9.50, or a brokerage of $\frac{1}{4}$%, for buying Union Pacific stock. How many shares did he purchase? 38.

7. A mining company declares a dividend of 15%. How much does Mr. Jones receive who owns 80 shares of stock? $1200.

8. I own 35 shares of bank stock. If the bank declares a dividend of 4%, how much shall I receive? **$140.**

9. A man pays an assessment of $7\frac{1}{2}$%, or $300, on his insurance stock. How many shares does he own? 40.

10. A man owns 60 shares of railroad stock. If the company declares a dividend of 5%, payable in stock, how much stock will he then own? 63 shares.

11. A gas company has a capital stock of $160000. Its gross earnings are $15700, and its expenses $4500 annually. What per cent can it pay the stockholders? 7%.

12. What will be the cost of 150 shares ($50 each) of a stock at $139\frac{3}{4}$, brokerage $\frac{1}{4}$%? $10500.

13. I bought 80 shares of American sugar, at 110, brokerage $\frac{1}{8}$%. How much did it cost me? $8810.

14. My broker sells 50 shares of Denver and Rio Grande, brokerage $\frac{1}{4}$%? He remits me $2475. At what price did the stock sell? $49\frac{3}{4}$.

15. What will be the cost of 25 Union Pacific 4's at $106\frac{1}{4}$, brokerage $\frac{1}{8}$%? $2659.375.

16. I paid $1560 for Colorado Southern at $19\frac{1}{4}$, brokerage $\frac{1}{4}$%. How many shares did I buy? 80.

17. How many shares of railroad stock at $104\frac{7}{8}$, brokerage $\frac{1}{8}$%, can be purchased for $4200? 40.

18. If I buy on one day 100 shares Mexican Central at $26\frac{7}{8}$, and sell them next day at $30\frac{1}{8}$, brokerage $\frac{1}{8}$% for buying and selling, how much do I gain? $300.

19. If I buy 100 shares of Iowa Central at $46\frac{7}{8}$, and sell at $32\frac{1}{8}$, brokerage $\frac{1}{8}$%, how much do I lose? $1500.

20. If I invest $39900 in 6% bonds at par, what will be my income? $2394.

21. If I invest \$39900 in 6% bonds at 105, what will be my income? \$2280.

22. If I invest \$39900 in 6% bonds at 95, what will be my income? \$2520.

23. How many shares of stock bought at $95\frac{1}{4}$ and sold at 105, brokerage $\frac{1}{4}$% on each transaction, will yield a profit of \$925? 100.

24. What must be paid for 6% bonds to realize an income of 8%? 75.

25. If I pay $87\frac{1}{2}$ for railroad bonds that yield an annual income of 7%, what per cent do I get on my investment? 8%.

26. What could I afford to pay for bonds yielding an annual income of 7% to invest my money so as to realize 6%? $116\frac{2}{3}$.

27. If I buy 5% bonds at 115, what income do I make on the investment? $4\frac{8}{23}$%.

28. How much must I pay for 5% bonds that the investment may yield 6% income? $83\frac{1}{3}$.

29. How much must I pay for Union Pacific 4's that the investment may yield 5%? 80.

30. If I invest \$4200 in 6% bonds at 105, what will be the rate of income on my investment? $5\frac{5}{7}$%.

31. What sum must be invested in 5% bonds at $98\frac{7}{8}$, brokerage $\frac{1}{8}$%, to secure an annual income of \$3720? \$73656.

32. What rate of income does stock paying 6% dividend yield if bought at 58? $10\frac{10}{29}$%.

33. What rate of income will a 6% bond bought at 110 yield? $5\frac{5}{11}$%.

34. What must I pay for Baltimore and Ohio $3\frac{1}{2}$'s to yield an income of 6% on the investment? $58\frac{1}{3}$.

INTEREST

181. **Interest** is money paid for the use of money.

NOTE.—The interest is paid by the *borrower* to the *lender.*

The **principal** is the money for the use of which interest is paid.

The **amount** is the sum of the principal and interest.

A **note** or a **promissory note** is a written promise to pay a certain sum of money on demand or at a specified time.

NOTE.—The borrower always gives the lender his note for the money. The following is a common form:

$500.00. DAYTON, O., *June* 16, 1903.

One year after date I promise to pay Charles Thomas, or order, five hundred dollars, with interest at 6%, for value received. JAMES Q. DEAN.

NOTE.—When a note is made to draw interest from date, the words "from date" are frequently inserted after the word "interest."

The **face** of the note or the sum promised to be paid is the principal.

Legal interest is interest at a per cent that is allowed by law.

NOTE.—When no rate of interest is mentioned, the legal rate, varying in different states from 5% to 8%, may be collected. In some states any interest may be collected if it is mentioned in the note; but in other states a limit is established, any excess over which is illegal.

SIMPLE INTEREST

182. 1. **Simple interest** is interest on the principal only.

NOTE.—Simple interest is not due and cannot be collected till the principal is due, unless so specified in the note.

2. In simple interest four quantities are considered, (1) the *principal*, (2) the *per cent*, (3) the *time*, and (4) the *interest*.

3. Any *three* of these quantities being given, the fourth may be found. There are *five cases*.

CASE I

183. Given the principal, the time, and the per cent, to find the interest:

1. Find the interest of $25 for 4 yr., at 6%.

SOLUTION.—For 1 yr. the interest is 6% of $25, or $1.50. For 4 yr. it is 4 × $1.50, or $6.00.

Or

For 1 yr. the interest is 6% of the principal. For 4 yr. it is 4 × 6%, or 24% of the principal. 24% of $25 = $6.

OPERATION

$25		$25
.06		.24
$1.50	or	100
4		50
$6.00 *Ans.*		$6.00 *Ans.*

2. Find the interest of $630 for 3 yr. 6 mo., at 5%.

SOLUTION.—The interest for 1 yr. is 5% of $630, or $31.50. 3 yr. 6 mo. = $3\frac{1}{2}$ yr. The interest for $3\frac{1}{2}$ yr. is $3\frac{1}{2}$ × $31.50, or $110.25.

OPERATION

$630
.05
31.50
$3\frac{1}{2}$
1575
9450
$110.25

3. Find the interest of $240 for 1 yr. 9 mo., at 6%.

OPERATION

$$\begin{array}{r} \$240 \\ .06 \\ \hline 14.40 \\ 1\frac{3}{4} \\ \hline 1080 \\ 1440 \\ \hline \$25.20 \end{array}$$

SOLUTION.—The interest of $240 for 1 yr., at 6% = $14.40. 1 yr. 9 mo. = $1\frac{3}{4}$ yr. $1\frac{3}{4} \times \$14.40 =$ $25.20, the interest for 1 yr. 9 mo.

4. Find the interest of $360 for 3 yr. 6 mo. 20 da., at 5%.

OPERATION

$$\begin{array}{rl} \$360 & \\ .05 & \\ \hline \$18.00 & = \text{Int. for 1 yr.} \\ 3\frac{5}{9} & \\ \hline 10 & \\ 54 & \\ \hline \$64 & = \text{Int. for 3 yr. 6 mo. 20 da.} \end{array}$$

SOLUTION.—The interest of $360 for 1 yr., at 5% = $18. 3 yr. 6 mo. 20 da. = 3 yr. 200 da. = $3\frac{5}{9}$ yr. $3\frac{5}{9} \times \$18 = \$64$, the interest for 3 yr. 6 mo. 20 da.

5. Find the amount of $630 for 3 yr. 6 mo., at 5%.

SOLUTION.—The interest of $630 for 3 yr. 6 mo., at 5%, is $110.25 (Ex. 2); the amount is $630 + $110.25, or $740.25.

OPERATION

$630 + $110.25 = $740.25

NOTE.—In computing interest it is customary to regard 30 days as 1 month, and 360 days as 1 year.

Rule.—1. *To find the interest, multiply the principal by the rate, and multiply the product by the time expressed in years.*

2. *To find the amount, add the principal and the interest.*

Find the interest of:

6.	\$65 for 4 yr., at 5%.	\$13.
7.	\$300 for 2 yr., at 6%.	\$36.
8.	\$275 for 3 yr., at 6%.	\$49.50.
9.	\$187.50 for 4 yr., at 5%.	\$37.50.

Find the amount of:

10.	\$45 for 2 yr., at 8%.	\$52.20.
11.	\$80 for 4 yr., at 7%.	\$102.40.
12.	\$237.16 for 2 yr., at $3\frac{3}{4}$%.	\$254.95.
13.	\$74.75 for 5 yr., at 4%.	\$89.70.
14.	\$85.45 for 4 yr., at 6%.	\$105.96.
15.	\$325 for 3 yr., at $5\frac{3}{8}$%.	\$377.65.
16.	\$129.36 for 4 yr., at $4\frac{3}{8}$%.	\$152.
17.	\$8745 for 2 yr., at 8%.	\$10144.20.

Find the interest of:

18.	\$300 for 1 mo., at 6%.	\$1.50.
19.	\$240 for 2 mo., at 8%.	\$3.20.
20.	\$50 for 5 mo., at 6%.	\$1.25.
21.	\$86 for 3 mo., at 6%.	\$1.29.
22.	\$50 for 4 mo., at 8%.	\$1.33.

Find the amount of:

23.	\$150.25 for 6 mo., at 8%.	\$156.26.
24.	\$360 for 7 mo., at 5%.	\$370.50.
25.	\$204 for 11 mo., at 7%.	\$217.09.
26.	\$228 for 9 mo., at 6%.	\$238.26.
27.	\$137.50 for 8 mo., at 6%.	\$143.00.
28.	\$7596 for 10 mo., at 8%.	\$8102.40.

Find the interest of:

29. $360 for 20 da., at 6%. $1.20.
30. $726 for 10 da., at 6%. $1.21.
31. $1200 for 15 da., at 6%. $3.00.
32. $180 for 19 da., at 8%. 76¢.
33. $240 for 27 da., at 7%. $1.26
34. $320 for 21 da., at 5%. 93¢.
35. $450 for 25 da., at 10%. $3.13.

Find the amount of:

36. $100.80 for 28 da., at 5%. $101.19.
37. $150 for 18 da., at 5%. $150.38.
38. $360 for 11 da., at 6%. $360.66.
39. $264 for 9 da., at 6%. $264.40.
40. $900 for 14 da., at 7%. $902.45.
41. $430 for 19 da., at 4½%. $431.02.

Find the interest of:

42. $150 for 4 yr. 2 mo., at 6%. $37.50.
43. $375.40 for 1 yr. 8 mo., at 6%. $37.54.
44. $92.75 for 3 yr. 5 mo., at 6%. $19.01.
45. $500 for 1 yr. 1 mo. 18 da., at 6%. $34.00.
46. $560 for 2 yr. 4 mo. 15 da., at 8%. $106.40.
47. $750 for 4 yr. 3 mo. 6 da., at 6%. $192.00.
48. $456 for 3 yr. 5 mo. 18 da., at 5%. $79.04.
49. $216 for 5 yr. 7 mo. 27 da., at 10%. $122.22.
50. $380 for 3 yr. 9 mo. 9 da., at 15%. $215.18.

Find the amount of:

51. \$300 for 3 yr. 8 mo., at 6%. \$366.00.

52. \$250 for 1 yr. 7 mo., at 6%. \$273.75.

53. \$205.25 for 2 yr. 8 mo. 15 da., at 6%. \$238.60.

54. \$150.62 for 3 yr. 5 mo. 12 da., at 5%. \$176.60.

55. \$210.25 for 2 yr. 7 mo. 20 da., at 7%. \$249.09.

56. \$57.85 for 2 yr. 3 mo. 23 da., at 5%. \$64.54.

57. Find the interest of \$150, from January 9, 1900, to April 19, 1902, at 6%. \$20.50.

NOTE.—To find the time between two dates, see § 77.

58. Find the interest of \$240, from February 15, 1902, to April 27, 1903, at 8%. \$23.04.

59. Find the interest of \$180, from May 14, 1901, to August 28, 1903, at 7%. \$28.84.

60. Find the interest of \$137.50, from July 3 to November 27, at 9%. \$4.95.

61. Find the amount of \$125.40, from March 1 to August 28, at $8\frac{1}{2}$%. \$130.64.

62. Find the amount of \$234.60, from August 2, 1902, to March 9, 1903, at $5\frac{1}{4}$%. \$242.02.

63. Find the amount of \$153.80, from October 25, 1904, to July 24, 1905, at 5%. \$159.55.

The Six Per Cent Method

184. This method is convenient because of the ease with which the interest of \$1 can be computed, and because 6% is the legal rate of interest in many states. At 6%

The interest of \$1 for 1 yr. = \$.06.
The interest of \$1 for 1 mo. = \$.005.
The interest of \$1 for 1 da. = \$.000$\frac{1}{6}$.

The interest of \$1 at 6% is therefore *one half as many cents as there are months and one sixth as many mills as there are days.*

1. Find the interest of \$540 for 3 yr. 4 mo. 18 da., at 6%.

OPERATION

Int. \$1 for 3 yr.	= \$.18
Int. \$1 for 4 mo.	= .02
Int. \$1 for 18 da.	= .003
Int. \$1 for 3 yr. 4 mo. 18 da.	= \$.203

$$\begin{array}{r} \$540 \\ .203 \\ \hline 1620 \\ 1080 \\ \hline \$109.62 \end{array}$$, *Ans.*

SOLUTION. — The interest of \$1 for 3 yr. @ 6% is \$.18; for 4 mo., \$.02; for 18 da., \$.003; for the whole time, \$.203. Since \$1 yields \$.203, the interest for the whole time is .203 of the principal. .203 of \$540 = \$109.62.

NOTE. — To find, by this method, the interest at any other rate, take such a fractional part of the interest at 6% as the given rate is part of 6. At 5% take $\frac{5}{6}$, since 5 is $\frac{5}{6}$ of 6, or subtract $\frac{1}{6}$. At 7% take $\frac{7}{6}$ or add $\frac{1}{6}$. At 8% take $\frac{4}{3}$ or add $\frac{1}{3}$. At $4\frac{1}{2}$% take $\frac{3}{4}$, since $4\frac{1}{2}$ is $\frac{3}{4}$ of 6.

2. Find the interest of \$540 for 3 yr. 4 mo. 18 da., at 5%. At 8%.

OPERATION

6)\$109.62
\$18.27
\$91.35, int. at 5%.

3)\$109.62
\$36.54
\$146.16, int. at 8%.

SOLUTION. — The interest at 6% was found in Ex. 1 to be \$109.62. At 5% it is $\frac{1}{6}$ less. Therefore subtract $\frac{1}{6}$. At 8% it is $\frac{1}{3}$ more. Therefore add $\frac{1}{3}$.

Rule. — *Reduce years to months. Take $\frac{1}{2}$ as many hundredths of the principal as there are months, and $\frac{1}{6}$ as many thousandths as there are days.*

Find by the 6% method the interest of $1:

3.	For 7 mo. 24 da., at 6%.	$0.039.
4.	For 10 mo. 15 da., at 5%.	$0.043¾.
5.	For 11 mo. 18 da., at 9%.	$0.087.
6.	For 1 yr. 2 mo. 9 da., at 6%.	$0.071½.
7.	For 2 yr. 5 mo. 12 da., at 8%.	$0.196
8.	For 3 yr. 10 mo. 17 da., at 10%.	0.388\frac{1}{18}$.
9.	For 4 yr. 3 mo. 11 da., at 7%.	0.299\frac{23}{36}$.
10.	For 5 yr. 7 mo. 24 da., at 4%.	$0.226.

Find by the 6% method the interest of:

11.	$40, for 6 mo. 21 da., at 6%.	$1.34.
12.	$50, for 8 mo. 24 da., at 9%.	$3.30.
13.	$120, for 10 mo. 12 da., at 7%.	$7.28.
14.	$200, for 11 mo. 15 da., at 6%.	$11.50.
15.	$500, for 1 yr. 3 mo. 6 da., at 3%.	$19.
16.	$750, for 1 yr. 5 mo. 27 da., at 8%.	$89.50.
17.	$48.75, for 1 yr. 9 mo. 3 da., at 6%.	$5.14.
18.	$76.32, for 1 yr. 10 mo. 25 da., at 4%.	$5.81.

Find the amount of:

19.	$600, for 2 yr. 1 mo. 9 da., at 5%.	$663.25.
20.	$900, for 2 yr. 4 mo. 10 da., at 6%.	$1027.50.
21.	$86.25, for 2 yr. 7 mo. 17 da., at 9%.	$106.67.
22.	$450, for 3 yr. 2 mo. 13 da., at 8%.	$565.30.
23.	$534.78, for 3 yr. 5 mo. 22 da., at 4%.	$609.17.
24.	$1200, for 3 yr. 11 mo. 15 da., at 10%.	$1675.

CASE II

185. Given the principal, the per cent, and the interest, to find the time:

1. The interest of $225 for a certain time, at 4%, was $66. What was the time?

OPERATION

$$\begin{array}{r} \$225 \\ .04 \\ \hline \$9.00 \end{array} \qquad \begin{array}{r} 9\underline{)66} \\ 7\tfrac{1}{3} \end{array}$$

$7\tfrac{1}{3}$ yr. = 7 yr. 4 mo.

SOLUTION. — The interest of $225 for 1 yr., at 4%, is $9; then, $66 will be the interest for as many years as 9 is contained times in 66, which is $7\tfrac{1}{3}$, or 7 yr. 4 mo.

2. In what time, at 10%, will $500 amount to $800?

OPERATION

$$\begin{array}{r} \$800 \\ 500 \\ \hline \$300 \\ \hline\hline \$500 \\ .10 \\ \hline \$\ 50)\$300 \\ 6 \end{array}$$

SOLUTION. — The interest will be $800 − $500 = $300. The interest of $500 for 1 yr., at 10%, is $50; then, $300 will be the interest for as many years as $50 is contained times in $300, which is 6.

3. In what time, at 8%, will any principal double itself?

OPERATION

$$\begin{array}{r} 8\underline{)100} \\ 12\tfrac{1}{2} \end{array}$$

$12\tfrac{1}{2}$ yr. = 12 yr. 6 mo.

SOLUTION. — A principal has doubled itself when the interest becomes 100%. Since the interest is 8% in 1 yr., it will be 100% in as many years as 8 is contained times in 100, which is $12\tfrac{1}{2}$, or 12 yr. 6 mo.

Rule. — *Divide the given interest by the interest of the principal for one year.*

NOTES. — 1. If the principal and amount are given, subtract the principal from the amount to find the interest.

2. If there is a fractional part of a year in the result, reduce it to months and days.

4. I lent $200, at 6%, and received $36 interest. How long was the money lent? 3 yr.

5. In what time, at 5%, will $60 amount to $72? 4 yr.

6. In what time, at 6%, will any principal be doubled? 16 yr. 8 mo.

7. A man lent $375, at 8%, and received $90 interest. How long was it lent? 3 yr.

8. In what time, at 9%, will $600 amount to $798? 3 yr. 8 mo.

9. In what time, at 10%, will any principal double itself? 10 yr.

10. How long will it take $250, at 6%, to yield $34.50 interest? 2 yr. 3 mo. 18 da.

11. How long will it take $60, at 6%, to amount to $73.77? 3 yr. 9 mo. 27 da.

12. How long will it take any principal to treble itself, at 6%? 33 yr. 4 mo.

13. The interest on $400, at 7%, was $68.60. How long was the principal loaned? 2 yr. 5 mo. 12 da.

14. In what time, at 9%, will $700 amount to $924.70? 3 yr. 6 mo. 24 da.

15. How long will it take any principal to increase one half, at 8%? 6 yr. 3 mo.

16. In what time, at 10%, will $1200 amount to $1675? 3 yr. 11 mo. 15 da.

17. In what time, at 4%, will $1500 double itself? 25 yr.

CASE III

186. **Given the principal, the time, and the interest, to find the per cent:**

1. A merchant paid $30 interest for the use of $300, for 1 yr. 8 mo. What was the per cent?

OPERATION

1 yr. 8 mo. $= \frac{5}{3}$ yr.

$\frac{30}{1} \times \frac{3}{5} = 18$

$\frac{18}{300} = \frac{3}{50}$

$\frac{3}{50} = 6\%$

SOLUTION. — 1 yr. 8 mo. are $1\frac{2}{3}$, or $\frac{5}{3}$ yr. Since the interest for $\frac{5}{3}$ yr. is $30, the interest for 1 yr. is $18. $18 is $\frac{3}{50}$ of $300; $\frac{3}{50} = 6\%$ (§ **162**).

2. At what per cent will any principal double itself in 20 yr.?

SOLUTION. — Since the interest for 20 yr. is 100%, the interest for 1 yr. is $\frac{1}{20}$ of 100% = 5%.

OPERATION

$100 \div 20 = 5$

Rule. — *Find the interest for one year, and find what per cent this is of the principal.*

3. I borrowed $600 for 2 years and paid $48 interest. What per cent did I pay? 4%.

4. A broker paid $200 interest for the use of $1000 for 2 yr. 6 mo. What was the per cent? 8%.

5. The amount of $250 for 2 yr. 4 mo. 24 da. was $310. What was the per cent? 10%.

6. $23.40 interest was paid for the use of $260 for 2 yr. What was the per cent? $4\frac{1}{2}\%$.

7. At what per cent will any principal double itself in 12 yr. 6 mo.? 8%.

8. The amount of $175 for 3 yr. 7 mo. was $250.25. What was the per cent? 12%.

9. The interest of $450 for 1 yr. 8 mo. 12 da. is $61.20. What is the per cent? 8%.

10. At what per cent will any principal double itself in 11 yr. 1 mo. 10 da.? 9%.

11. The amount of $650 for 2 yr. 5 mo. 18 da. is $746.20. What is the per cent? 6%.

12. The interest of $640 for 6 yr. was $110.40. What was the per cent? $2\frac{7}{8}$%.

CASE IV

187. Given the time, per cent, and interest, to find the principal:

1. The interest for 3 yr., at 9%, is $21.60. What is the principal?

OPERATION

$9\% = .09$

$.09 \times 3 = .27$

$\$21.60 \div .27 = \$80$

SOLUTION.—9% is .09. Since the interest for 1 yr. is .09 of the principal, for 3 yr. it is .27 of the principal. Then, the principal, multiplied by .27, is $21.60, and the principal is $21.60 ÷ .27 = $80.

2. The interest for 2 yr., at 6%, is $27. What is the principal?

OPERATION

$6\% = \frac{3}{50}$

$\frac{3}{50} \times \frac{2}{1} = \frac{3}{25}$

$\frac{\overset{9}{\cancel{27}}}{1} \times \frac{25}{\cancel{3}} = 225$

SOLUTION.—6% is $\frac{3}{50}$. Since the interest for 1 yr. is $\frac{3}{50}$ of the principal, for 2 yr. it is $\frac{3}{25}$ of the principal. Then, $\frac{3}{25}$ of the principal = $27, $\frac{1}{25}$ of the principal is $9, and the principal is $225.

Rule.—*Multiply the rate by the time, and divide the interest by the product.*

3. The interest for 3 yr., at 5%, is $8.25. What is the principal? $55.

4. The interest for 3 yr., at 5%, is $341.25. What is the principal? $2275.

5. The interest for 1 yr. 4 mo., at 6%, is $2.26. What is the principal? $28.25.

6. What principal, at 5%, will produce a yearly interest of $1023.75? $20475.

7. What principal, at 8%, will produce $30.24 interest in 1 yr. 6 mo. 27 da.? $240.

8. What principal, at 9%, will produce $525.40 interest in 12 yr. 3 mo. 20 da.? $474.40.

9. The interest for 2 yr. 7 mo. 11 da., at 4%, is $9.41. What is the principal? $90.

10. The interest for 5 yr. 8 mo. 24 da., at 6%, is $28.38. What is the principal? $82.50.

CASE V

188. Given the time, per cent, and amount, to find the principal:

1. What principal in 5 yr., at 6%, will amount to $650?

Solution. — The interest of $1 for 5 yr. at 6% is $.30 and the amount is $1.30. Therefore, $650 is the amount of as many dollars as $1.30 is contained in $650, that is of $500.

OPERATION

$5 \times \$.06 = \$.30$

$\$1 + \$.30 = \$1.30$

$\$1.30 \,|\, \$650$

$\$500$

Rule. — *Multiply the rate by the time, and divide the amount by 1 + the product.*

2. What principal in 9 yr., at 5%, will amount to $435? $300.

3. The amount for 4 yr., at 5%, is $571.20. What is the interest? $95.20.

4. The amount for 6 yr., at 7%, is \$532.50. What is the interest? \$157.50.

5. The amount for 2 yr. 9 mo., at 8%, is \$285.48. What is the principal? \$234.

6. The amount for 2 yr. 6 mo., at 6%, is \$690. What is the interest? \$90.

7. The amount for 3 yr. 4 mo. 24 da., at 7%, is \$643.76. What is the principal? \$520.

8. The amount for 4 yr. 3 mo. 27 da., at 4%, is \$914.94. What is the interest? \$134.94.

189. Formulas for the five cases of Interest:

Let p represent the principal, t the time, r the rate, and i the interest. Then,

Case I. $p \times r \times t = i.$

Case II. $i \div (p \times r) = t.$

Case III. $(i \div t) \div p = r.$

Case IV. $i \div (r \times t) = p.$

Case V. $\dfrac{p + i}{1 + (r \times t)} = p.$

COMPOUND INTEREST

190. In **compound interest** the principal is increased yearly by the addition of the interest.

NOTES.—1. Sometimes the interest is added semiannually, or quarterly.

2. The way in which interest is legally compounded is, at the end of each year, to take up the old note and give a new one with a face equal to both the principal and interest of the former note.

1. Find the compound interest of \$300 for 3 yr., at 6%.

SOLUTION. — The interest of \$300 for 1 yr., at 6%, is \$18; the amount is \$18 + \$300 = \$318. The interest of \$318 for 1 yr., at 6%, is \$19.08; the amount is \$19.08 + \$318 = \$337.08. The interest of \$337.08 for 1 yr., at 6%, is \$20.2248; the amount is \$20.2248 + \$337.08 = \$357.3048.

The compound interest is \$357.3048 − \$300 = \$57.30.

OPERATION

$$\begin{array}{r} \$300 \\ .06 \\ \hline \$18.00 \\ 300 \\ \hline \$318. \\ .06 \\ \hline \$19.08 \\ 318 \\ \hline \$337.08 \end{array} \qquad \begin{array}{r} \$337.08 \\ .06 \\ \hline \$20.2248 \\ 337.08 \\ \hline \$357.3048 \\ 300 \\ \hline \$57.3048 \end{array}$$

Rule. — 1. *Find the amount of the given principal for the first year, and make it the principal for the second year.*

2. *Find the amount of this principal for the second year, make it the principal for the third year, and so on for the given number of years.*

3. *From the last amount subtract the given principal; the remainder will be the compound interest.*

NOTES. — 1. When the interest is payable half-yearly, or quarterly, find the interest for a half, or a quarter year, and proceed in other respects as when the interest is payable yearly.

2. When the time is years, months, and days, find the amount for the years, then compute the interest on this for the months and days, and add it to the last amount.

Find the amount, at 6%, compound interest, of:

2. \$500, for 3 years. \$595.51.

3. \$800, for 4 years. \$1009.98.

Find the compound interest of:

4. \$250 for 3 yr., at 6%. \$47.75.

5. \$300 for 4 yr., at 5%. \$64.65.

6. Find the compound interest of \$200 for 2 yr., at 6%, payable semiannually. \$25.10.

7. Find the amount of \$500 for 2 yr., at 20% compound interest, payable quarterly. \$738.73.

8. What is the compound interest of \$300 for 2 yr. 6 mo., at 6%? \$47.19.

9. What is the amount of \$620 at compound interest semiannually, for 3 yr. 6 mo., at 6%? \$762.52.

10. Find the difference between simple interest and compound interest on \$500 for 4 yr. 8 mo., at 6%. \$16.49.

ANNUAL INTEREST

191. Annual interest is interest on the principal, and on each annual interest after it is due.

NOTE.—Annual interest may be collected when the note or bond reads "with interest payable annually."

1. No interest having been paid, find the amount due in 4 yr. 8 mo. 24 da. on a note for \$400, with interest at 6%, payable annually.

SOLUTION.—The interest of \$400 for 4 yr. 8 mo. 24 da., at 6%, is \$113.60. One annual interest of \$400, at 6%, is \$24. The first annual interest remains unpaid 3 yr. 8 mo. 24 da.; the second, 2 yr. 8 mo. 24 da.; the third, 1 yr. 8 mo. 24 da.; and the fourth, 8 mo. 24 da.; hence interest must be reckoned on \$24 for 8 yr. 11 mo. 6 da.; this is \$12.864. The amount then is \$12.864 + \$113.60 + \$400 = \$526.46.

OPERATION

\$.24 + \$.04 + \$.004 = \$.284, int. of \$1 for 4 yr. 8 mo. 24 da.

$$\begin{array}{r} \$.284 \\ 400 \\ \hline \$113.600 \end{array} \qquad \begin{array}{r} \$400 \\ .06 \\ \hline \$24.00 \end{array}$$

\$.48 + \$.055 + \$.001 = \$.536, int. of \$1 for 8 yr. 11 mo. 6 da.

yr.	mo.	da.
3	8	24
2	8	24
1	8	24
	8	24
8	11	6

$$\begin{array}{r} \$.536 \\ 24 \\ \hline 2144 \\ 1072 \\ \hline \$12.864 \\ 113.60 \\ 400. \\ \hline \$526.464 \end{array}$$

Rule. — 1. ***Find the interest of the principal for the time during which no annual interest is paid.***

2. ***Find the interest of one annual interest for the sum of the times each annual interest remains unpaid.***

3. ***The sum of the two interests will be the interest due, and this, added to the principal, will be the amount due.***

2. No interest having been paid, find the amount due in 3 yr. on a note for $800, with interest at 8%, payable annually. $1007.36.

3. No interest having been paid for 4 yr., find the interest due on a bond for $10000, with interest at 5%, payable annually. $2150.

4. No interest having been paid, find the amount due Sept. 1, 1903, on a note for $500, dated June 1, 1901, with interest at 6%, payable semiannually. $571.10.

5. [$1200.] MILWAUKEE, WIS., May 12, 1903.

For value received, on demand, I promise to pay John G. Morgan, or order, twelve hundred dollars, with interest at 6%, payable annually. H. W. SLOCUM.

No interest having been paid, what is the amount due on this note Sept. 20, 1907? $1545.66.

6. [$1500.] NEW ORLEANS, LA., Oct. 10, 1896.

On the first day of May, 1901, for value received, I promise to pay Andrew Jackson, or order, fifteen hundred dollars, with interest, payable annually, at 5%.

GEORGE QUITMAN.

No interest having been paid, what amount was due at maturity? $1872.75.

7. What is the difference between simple and annual interest on $1000 for 5 yr. at 6%? $36.

PARTIAL PAYMENTS

192. A **partial payment** is a sum of money, less than the face, paid on a note (§ **181**).

The receipt of a partial payment is acknowledged by **indorsing** it on the back of the note.

The **indorsement** consists of the date and amount of the payment.

The rule announced by Chancellor Kent with reference to partial payments, is as follows :

"*When partial payments have been made, apply the payment, in the first place, to the discharge of the interest then due.*

"*If the payment exceeds the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due.*

"*If the payment is less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal, until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal; and interest is to be computed on the balance, as aforesaid.*" — KENT.

NOTE. — This rule is founded on the principle that neither interest nor payment shall draw interest.

1. [$1000.] BOSTON, MASS., May 1, 1901.

For value received, on demand, I promise to pay to Alonzo Warren, or order, one thousand dollars, with interest at 6%. WILLIAM MURDOCK.

On this note partial payments were indorsed as follows: November 25, 1901, $134; March 7, 1902, $315.30; August 13, 1902, $15.60; June 1, 1903, $25; April 25, 1904, $236.20. What was the amount due on settlement, September 10, 1904?

Solution. — The time from May 1, 1901, to November 25, 1901, is 6 mo. 24 da.; the interest of $1 for this time is $.034, and the interest of $1000 is $34; the payment, $134, *exceeds* the interest; the amount is $1034; $1034 – $134 = $900, *the second principal.*

The time from November 25, 1901, to March 7, 1902, is 3 mo. 12 da.; the interest of $900 for this time is $15.30; the payment, $315.30, *exceeds* the interest; the amount is $915.30; $915.30 – $315.30 = $600, *the third principal.*

The time from March 7, 1902, to August 13, 1902, is 5 mo. 6 da.; the interest of $600 for this time is $15.60; the payment, $15.60, *equals* the interest; the amount is $615.60; $615.60 – $15.60 = $600, *the fourth principal.*

The time from August 13, 1902, to June 1, 1903, is 9 mo. 18 da.; the interest of $600 for this time is $28.80; the payment, $25, *is less* than the interest; continue to find the interest on $600, *the fourth principal.*

The time from June 1, 1903, to April 25, 1904, is 10 mo. 24 da.; the interest of $600 for this time is $32.40; the sum of the payments, $261.20, *exceeds* the sum of the interests, $61.20; the amount is $661.20; $661.20 – $261.20 = $400, *the fifth principal.*

OPERATION

1901	11	25	$134	$1000
1901	5	1		.034
	6	24	.034	34
				1000
				$1034
				134
1902	3	7	$315.30	$900
1901	11	25		.017
	3	12	.017	$15.30
				900
				$915.30
				315.30
1902	8	13	$15.60	$600
1902	3	7		.026
	5	6	.026	$15.60
				600
				$615.60
				15.60
1903	6	1	$25	$600
1902	8	13		.048
	9	18	.048	$28.80
				$600
			$25	.054
1904	4	25	236.20	$32.40
1903	6	1	$261.20	28.80
	10	24	.054	$61.20
				600
				$661.20
				261.20
1904	9	10		$400
1904	4	25		.0225
	4	15	.0225	$9
				400
				$409

The time from April 25, 1904, to September 10, 1904, is 4 mo. 15 da.; the interest of $400 for this time is $9; the amount due on settlement is $409.

I. When each payment equals or exceeds the interest:

Rule. — 1. *Find the time from the date of the note to the date of the first payment.*

2. *Find the amount of the given principal for this time.*

3. *From this amount subtract the payment; the remainder is the second principal.*

4. *Find the time from the date of the first payment to the date of the second payment.*

5. *Then proceed with the second principal as with the first, and so on to the date of settlement.*

II. When one or more payments are less than the interest:

Rule. — 1. *Continue to find the interest on the same principal until a date is reached when the sum of the payments equals or exceeds the sum of the interests.*

2. *Then subtract the sum of the payments from the amount; the remainder is the next principal.*

NOTE. — Sometimes it may be determined, by inspection, that the payment is less than the interest; when this can be done, it is not necessary to find the intermediate time and interest, but interest may at once be found to the date when it is apparent that the sum of the payments exceeds the interest.

2. [$350.] BOSTON, MASS., July 1, 1901.

For value received, I promise to pay Edward Sargent, or order, on demand, three hundred and fifty dollars, with interest at 6%. JAMES GORDON.

Indorsements: March 1, 1902, \$44; October 1, 1902, \$10; January 1, 1903, \$26; December 1, 1903, \$15. What was the amount due on settlement, March 16, 1904? \$306.75.

3. A note of \$200 is dated January 1, 1900. Indorsement: January 1, 1901, \$70. What was the amount due January 1, 1902, interest at 6%? \$150.52.

4. A note of \$300 is dated July 1, 1901. Indorsements: January 1, 1902, \$109; July 1, 1902, \$100. What was the amount due January 1, 1903, interest at 6%? \$109.18.

5. A note of \$150 is dated May 10, 1902. Indorsements: September 10, 1903, \$32; September 10, 1904, \$6.80. What was the amount due November 10, 1904, interest at 6%? \$132.30.

6. A note of \$200 is dated March 5, 1898. Indorsements: June 5, 1899, \$20; December 5, 1899, \$50.50. What was the amount due June 5, 1901, interest at 10%? \$189.18.

7. A note of \$250 is dated January 1, 1901. Indorsements: June 1, 1901, \$6; January 1, 1902, \$21.50. What was the amount due July 1, 1902, interest at 7%? \$248.40.

8. A note of \$180 is dated August 1, 1900. Indorsements: February 1, 1901, \$25.40; August 1, 1901, \$4.30; January 1, 1902, \$30. What was the amount due July 1, 1902, interest at 6%? \$138.54.

9. A note of \$400 is dated March 1, 1897. Indorsements: September 1, 1897, \$10; January 1, 1898, \$30; July 1, 1898, \$11; September 1, 1898, \$80. What was the amount due March 1, 1899, interest at 6%? \$313.33.

10. A note of $450 is dated April 16, 1900. Indorsements: January 1, 1901, $20; April 1, 1901, $14; July 16, 1901, $31; December 25, 1901, $10; July 4, 1902, $18. What was the amount due June 1, 1903, interest at 8%? $466.50.

193. When partial payments are made on notes and accounts running a year or less, the amount due is commonly found by the mercantile rule.

Mercantile Rule. — 1. *Find the amount of the principal from the date of the note to the date of settlement.*

2. *Find the amount of each payment from its date to the date of settlement.*

3. *From the amount of the principal, subtract the sum of the amounts of the payments.*

1. A note of $320 is dated Jan. 1, 1901. Indorsements: May 1, 1901, $50; Nov. 16, 1901, $100. What was the amount due Jan. 1, 1902, interest at 6%? $186.45.

2. An account of $540 was due March 1, 1902. Credits: May 1, 1902, $90; July 1, 1902, $100; Aug. 1, 1902, $150; Oct. 11, 1902, $180. What was the amount due on settlement Jan. 1, 1903, interest at 8%? $39.

BANK DISCOUNT

194. A **bank** is an institution chartered under the law for the purpose of receiving and loaning money and issuing bank bills.

Bank discount is a deduction made by a bank from the face of a note for cashing it before it is due. It is simple interest collected in advance.

The **term of discount** is the number of months or days from the date the note is discounted to the time it legally matures.

NOTE. — To find when a note matures if the time is expressed in days, count the exact number of days from the date of the note. In a common year a sixty-day note dated February 26 matures April 27, since 2 da. in February + 31 da. in March + 27 da. in April = 60 da. If the time is expressed in months — "two months after date" — count even months. February 26 + 2 mo. = April 26, the date of maturity. In some states three days of grace are allowed after the expiration of the time named in the note, but *in the examples in this book days of grace are not to be reckoned unless specified.* Notes that fall due on a Sunday or a legal holiday are in most states payable on the day after, but in some states on the day before.

The **proceeds** of a note is the face of the note less the bank discount.

195. The following are common forms of notes:

$500. CHICAGO, ILL., October 21, 1902.

Ninety days after date, we jointly and severally promise to pay to the Second National Bank of Chicago, Ill., five hundred dollars, for value received.

O. S. WEST.
W. B. SHARP.

Due January 19, 1903.

NOTE. — This is an accommodation note, payable to the bank which lends the money.

$200. BUFFALO, N.Y., March 21, 1901.

On demand, I promise to pay Charles H. Peck, two hundred dollars, for value received, with interest at 6%.

G. W. CLINTON.

NOTE. — This note is payable only to Charles H. Peck; it is due at once, and bears interest from date.

$1000. St. Louis, Mo., May 1, 1902.

One year after date I promise to pay to David King, or order, one thousand dollars, for value received.

Elmer B. Archer.

Note.—The words "or order" make this note negotiable. If David King transfers it, he must indorse it—that is, write his name across the back of it. This note bears no interest until after it is due.

$150. Washington, D.C. August 10, 1900.

On or before the first day of May, 1901, I promise to pay Amos Durand, or bearer, one hundred fifty dollars, with interest at 6% from date, for value received.

John Sherwood.

Note.—The words "or bearer" make this note negotiable without indorsement. This note bears interest from date, it being so specified.

In bank discount four quantities are considered: (1) The *face of the note*, (2) the *per cent*, (3) the *time*, and (4) the *discount*.

Any *three* of these quantities being given, the fourth may be found. We shall consider *two cases*.

CASE I

196. Given the face of the note, the per cent, and the time, to find the discount and the proceeds:

1st. *When the note does not bear interest:*

1. Find the date when due, bank discount, and proceeds of the following note, discounted at 6%:

$700. Cincinnati, Ohio, June 26, 1902.

Sixty days after date, we jointly and severally promise to pay to the First National Bank of Cincinnati, Ohio, seven hundred dollars for value received.

Charles Walker.
Walter Smith.

OPERATION

$$
\begin{array}{r}
6\underline{).060} \\
\$.01 \\
\underline{700} \\
\$7.00 \\
\\
\$700 \\
\underline{7} \\
\$693
\end{array}
$$

SOLUTION.—The note is due August 25, 1902 (§ 78). The interest of $1 for 60 days, at 6%, is $0.01, and the interest of $700 is $7; this is the discount; then, $700 − $7 = $693, the proceeds.

NOTE.—In a state where 3 days of grace are allowed this note matures 63 da. after June 26, or Aug. 28. The interest on $1 for 63 da. is $.0105; the interest on $700 is $7.35; and the proceeds is $700 − $7.35, or $692.65.

Rule. — 1. *Find the interest on the face of the note for the given time; this is the bank discount.*

2. *From the face of the note subtract the discount; the remainder is the proceeds.*

Find the date when due, bank discount, and proceeds of the following notes, if discounted the day they are dated:

2. A note of $100, dated June 20, payable in 60 days, and discounted at 6%. August 19, $1, $99.

3. A note of $120, dated October 12, payable in 30 days, and discounted at 8%.
November 11, $0.80, $119.20.

4. A note of $140, dated January 15, payable in 4 months, and discounted at 6%.
May 15, $2.80, $137.20.

5. A note of $180, dated April 10, payable in 6 months, and discounted at 4%.
October 10, $3.60, $176.40.

6. A note of \$250, dated December 1, payable in 5 months, and discounted at 8%.

May 1, \$8.33, \$241.67.

7. A note of \$375, dated August 4, payable in 30 days, and discounted at 6%.

September 3, \$1.88, \$373.12.

8. A note of \$600, dated February 12, 1904, payable in 60 days, and discounted at 9%. (Allow grace.)

April 12/15, \$9.45, \$590.55.

9. A note of \$1200, dated February 20, 1902, payable in 90 days, and discounted at 10%.

May 21, \$30, \$1170.

10. A note of \$1780, dated January 12, 1904, payable in 90 days, and discounted at 6%.

April 11, \$26.70, \$1753.30.

Find the date when due, time of discount, bank discount, and proceeds of the following notes:

11. [\$600.] TACOMA, WASH., Sept. 15, 1904.

One year after date, I promise to pay to the order of Abel E. Worth, at the First National Bank of San Francisco, Cal., six hundred dollars, for value received.

GEORGE M. BURGESS.

Discounted May 18, 1905, at 10%.

Sept. 15, 1905, 120 days, \$20, \$580.

12. [\$1000.] NASHVILLE, TENN., May 6, 1902.

Three months after date, I promise to pay Albert E. Kirk, or order, one thousand dollars, for value received.

JACOB SIMMONS.

Discounted June 19, 1902, at 6%.

August 6, 48 days, \$8, \$992.

2d. *When the note bears interest:*

1. *Find the date when due, time of discount, bank discount, and proceeds of the following business note:*

$800. BALTIMORE, MD., January 6, 1903.

Six months after date, I promise to pay to the order of Charles Stuart, at the Dayton National Bank, of Dayton, O., eight hundred dollars, with interest at 6%, for value received. FRANCIS MURPHY.

Discounted April 13, 1903, at 8%.

SOLUTION.—The note is due July 6, 1903. The time of discount, from April 13 to July 6, is 84 days. The amount of $800 for 6 mo., at 6%, is $824. The bank discount of $824 for 84 days, at 8%, is $15.38. The proceeds are $808.62.

OPERATION

$2\overline{).06}$
.03

$6\overline{).084}$
$.014 \times \frac{4}{3} = .018\frac{2}{3}$

$824.00
15.38
$808.62

$800
.03
24.00
800
$824
$.018\frac{2}{3}$
$549\frac{1}{3}$
6592
824
$\$15.381\frac{1}{3}$

Rule.—1. *Find the amount of the note for the given time.*

2. *Find the bank discount and proceeds of this amount.*

NOTE.—In the following examples, remember that in leap years February has 29 days.

Find the date when due, time of discount, bank discount, and proceeds of:

2. A note of $150, dated May 20, 1902, payable in 6 months, with interest at 6%, and discounted September 6, 1902, at 8%.

November 20, 1902, 75 days, $2.58, $151.92.

3. A note of $300, dated August 5, 1904, payable in 1 year, with interest at 8%, and discounted April 13, 1905, at 6%.

August 5, 1905, 114 days, $6.16, $317.84.

4. A note of $450, dated March 2, 1902, due in 10 months, with interest at 6%, and discounted August 11, 1902, at 10%.

January 2, 1903, 144 days, $18.90, $453.60.

5. A note of $650, dated May 1, 1896, due in 2 years 4 months, with interest at 9%, and discounted April 22, 1898, at 6%.

Sept. 1, 1898, 132 days, $17.30, $769.20.

6. A note of $840, dated September 1, 1903, payable in 6 months, with interest at 10%, and discounted December 17, 1903, at 8%.

March 1, 1904, 75 days, $14.70, $867.30.

7. A note of $1400, dated Aug. 1, 1903, due in 9 months, with interest at 6%, and discounted Jan. 17, 1904, at 10%.

May 1, 1904, 105 days, $42.67, $1420.33.

8. A note of $2400, dated Oct. 4, 1896, due in 1 year 3 months, with interest at 8%, and discounted July 26, 1897, at 10%.

January 4, 1898, 162 days, $118.80, $2521.20.

9. [$3500.] WINDSOR, CONN., Oct. 16, 1900.

One year after date, I promise to pay Adam Moore, or order, thirty-five hundred dollars, with interest at 6%, for value received. JOSEPH STEPHENS.

Discounted May 13, 1901, at 9%.

October 16, 1901, 156 days, $144.69, $3565.31.

10. [$6000.] Columbus, Ohio, May 11, 1895.
One year after date, I promise to pay Henry Warren, or order, six thousand dollars, with interest at 8%, for value received. Amos E. Burton.

Discounted November 19, 1895, at 10%.
May 11, 1896, 174 days, $313.20, $6166.80.

CASE II

197. Given the per cent, the time, and the proceeds, to find the face of the note:

1. For what sum due 90 days hence, must I give a note to a bank, that, when discounted at 6%, the proceeds will be $177.30?

OPERATION

$$6\underline{)\$.090}\qquad \$1.000$$
$$\$.015\qquad \underline{.015}$$
$$\$.985$$

$$\$.985)\$177.300(\$180$$
$$\underline{985}$$
$$7880$$
$$\underline{7880}$$
$$0$$

Solution.—The bank discount of $1 for 90 days, at 6%, is $0.015, and the proceeds $1 − $0.015 = $0.985. Then, $177.30 is the proceeds of 177.30 ÷ .985 = $180.

Rule.—1. *Find the proceeds of $1 for the given time at the given per cent.*

2. *By this divide the given proceeds.*

2. The proceeds of a note discounted at a bank for 60 days, at 6%, was $198. What was the face of the note? $200.

3. For what sum must a note be made, so that when discounted at a bank, for 90 days, at 6%, the proceeds will be $394? $400.

4. What must be the face of a note, that when discounted at a bank for 5 months, at 8%, the proceeds may be $217.50? $225.

5. The proceeds of a note is $352.80, the time 4 months, and the discount 6%. What is the face? $360.

6. I wish to borrow $400 from a bank for 30 days. What must be the face of my note, that, when discounted at 6%, I may receive this amount? $402.02.

7. I wish to obtain from a bank $500 for 60 days. For what sum must I give my note, at 8% discount? $506.76.

8. I wish to use $1500 for 6 months; if I can obtain money from a bank, at a discount of 10%, for what sum must I give my note to realize this amount? $1578.95.

PRESENT WORTH

198. The **present worth** of a debt is a sum of money, which, when put at interest for the given time at a given per cent, will amount to the debt when it becomes due.

NOTE.—The difference between a debt and its present worth is sometimes called **true discount**; but in business the only kinds of discount in use are trade discount and bank discount.

199. Given the face of the note, the time, and the per cent, to find the present worth.

1. Find the present worth, at 6%, of a note of $430.50, due in 2 yr. 5 mo. 18 da.

OPERATION

```
$1.148)$430.50($375
        3444
        ----
         8610
         8036
         ----
          5740
          5740
          ----
```

SOLUTION.—The amount of $1 for 2 yr. 5 mo. 18 da., at 6%, is $1.148. Then, the present worth of $430.50 is $430.50 ÷ $1.148 = $375.

2. Find the present worth, at 8%, of a note of $500, due in 3 yr., and bearing interest at 6%.

OPERATION

$500	$ .08
.06	3
$30.00	.24
3	1.00
$90	$1.24
500	
$590	

$1.24)$590($475.81
496
940
868
720
620
1000
992
80

SOLUTION.—The amount of $500 for 3 yr., at 6%, is $590. The amount of $1 for 3 yr., at 8%, is $1.24. Then, the present worth of $590 is $590 ÷ $1.24 = $475.81.

Rule.—1. *Find the amount of $1 for the given time at the given per cent.*

2. *By this divide the amount of the note; this is the present worth.*

NOTE.—When the note does not bear interest, of course the amount is the same as the face of the note.

3. Find the present worth, at 6%, of a note of $224, due in 2 yr. $200.

4. Find the present worth, at 6%, of a note of $300 due in 2 yr., and bearing interest at 8%. $310.71

5. Find the present worth, at 6%, of a debt of \$675, due in 5 yr. 10 mo. \$500.

6. Find the present worth for 5 mo., at 10%, of an account of \$368.75. \$354.

7. A merchant bought a bill of goods amounting to \$775, on 4 months' credit. If money is worth 10% to him, what might he pay for the goods in cash? \$750.

8. A man bought a bill of goods amounting to \$260, on 8 months' credit. If money is worth 6%, what sum will pay the debt in cash? \$250.

9. A merchant buys a bill of goods amounting to \$2480; he can have 4 months' credit, or 5% off for cash. If money is worth only 10% to him, what is his gain if he pays cash? \$44.

10. Find the present worth, at 5%, of a debt of \$956.34, one third to be paid in 1 yr., one third in 2 yr., and one third in 3 yr. \$870.60.

11. A man bought goods amounting to \$750, the bill payable in 3 months, or 4% allowed for cash. If money is worth 8% to him, how much will he gain by paying cash? \$15.29.

EXCHANGE

200. A **draft**, or a **bill of exchange**, is a written order, from one person to another, to pay a certain amount of money at a specified time to the party named in the order.

NOTES. — 1. The person upon whom the bill is drawn is called the **drawee**; the person in whose favor it is drawn is called the **payee**.

2. When the draft is to be paid upon presentation it is called a **sight draft**; when it is to be paid at the end of a certain time it is called a **time draft**.

3. An order on a bank by a person who has deposited money there is called a **check**.

Exchange is the method of making payments in distant places by means of a draft, or a bill of exchange.

There are two sorts of exchange, *domestic* and *foreign*.

DOMESTIC EXCHANGE

Domestic exchange takes place between localities in the same country.

NOTE. — When the banks of, say, Chicago have not enough money deposited in New York to meet their drafts on New York, they must send on money. The price in Chicago of drafts on New York will then be **at a premium**. But when the banks of Chicago have a large surplus deposited in New York it is desirable for them to sell drafts on New York, and drafts in Chicago on New York will be **at a discount**.

The following is a common form of a domestic bill of exchange, which is usually termed a *draft*.

$500. CINCINNATI, O., May 1, 1902.

At sight, pay to the order of Grover Parker, five hundred dollars, value received, and charge to account of

SILAS THOMPSON.

To Charles Smith & Co., Bankers, New York.

Foreign exchange takes place between localities in different countries.

The following is a common form of a foreign bill of exchange:

£500. CINCINNATI, O., May 1, 1902.

At sight of this Original of Exchange (Duplicate of the same tenor and date unpaid), pay to the order of Amos Carroll, Five hundred Pounds Sterling, value received, and charge to account of STANLEY BINGHAM.

To Baring Brothers, London.

A foreign bill of exchange is usually drawn in duplicate; the different copies, termed respectively the original and duplicate or *first* and *second* of exchange, are then sent by different mails, that miscarriage or delay may be avoided. When one is paid the other is void.

The **acceptance** of a bill of exchange is the agreement by the drawee to pay it when due.

NOTE.—A bill is accepted by the drawee's writing the word "accepted," with his name, across the face of the bill; the bill is then an acceptance.

201. To find the cost or face of a domestic bill of exchange (§ **170**, Rule):

NOTE.—Interest and premium are calculated on the face of the draft.

1. What is the cost of a sight draft on New York for $1400, at $\frac{1}{2}$% premium? $1407.

2. What is the cost of a sight draft on Boston for $2580, at $\frac{1}{2}$% discount? $2567.10.

3. What is the face of a sight draft on Wheeling, which cost $375.87, at $\frac{1}{8}$% premium? $375.40.

4. What is the cost of a sight draft on Chicago for \$2785, at $\frac{1}{4}\%$ discount? \$2778.04.

5. What is the face of a sight draft, which cost \$1852.55, at $1\frac{1}{4}\%$ discount? \$1876.

6. What is the cost of a draft on Boston for \$5680, payable in 60 days, exchange being at $\frac{1}{2}\%$ premium, and interest 6%? \$5651.60.

7. What is the cost of a draft on Baltimore for \$1575, payable in 30 days, exchange being at $\frac{3}{4}\%$ premium, and interest 6%? \$1578.94.

8. The face of a draft, payable in 60 days, is \$2625. Exchange being at $1\frac{1}{2}\%$ premium, and interest 6%, what is the cost of the draft? \$2638.12.

FOREIGN EXCHANGE

202. Foreign bills of exchange are drawn in the money of the country in which they are to be paid.

NOTE.—The foreign exchange of the United States is chiefly with Great Britain, France, Germany, Holland, and Canada.

ENGLISH MONEY

The unit of English money is the pound sterling.

TABLE

		U.S. Value
4 farthings	= 1 penny, marked d. =	\$0.0202.
12 pence	= 1 shilling, marked s. =	0.2433.
20 shillings	= 1 pound, marked £ =	4.8665.

NOTE.—The usual coins are: *gold*, sovereign = £1, and half sovereign; *silver*, crown = 5 s., half crown, florin = 2 s., shilling, sixpence, fourpence, threepence, twopence, and penny; *bronze*, the penny, halfpenny, and farthing.

FRENCH MONEY

The unit of French money is the franc, marked fr. ($0.193), which is divided into 100 centimes.

NOTE.—The usual coins are: *gold* pieces for 100, 50, 20, 10, and 5 francs; *silver* pieces for 5, 2, and 1 francs, and 50 and 20 centimes; *bronze* pieces for 10, 5, 2, and 1 centimes. The smallest coin in general use is the 5 centime piece, also called a *sou*.

GERMAN MONEY

The unit of German money is the mark (.238), which is divided into 100 pfennigs.

NOTE.—The usual coins are: *gold* pieces for 20 and 10 marks; *silver* pieces for 5, 2, and 1 marks, and 50 and 20 pfennigs; *nickel* pieces for 10 and 5 pfennigs; and *bronze* pieces for 2 and 1 pfennigs.

Canadian money is in dollars and cents, corresponding with United States currency.

The **par of exchange** is the comparative value of the standard coins of two countries.

NOTE.—The commercial value of foreign exchange fluctuates and may be above or below the par value.

The par value of the pound is $4.8665. The par value of the franc is $0.193, quoted as 5.18 francs to the dollar. The par value of the mark is $0.238, quoted as 4 marks to $.952.

203. To find the cost or face of a foreign bill of exchange:

1. How much will a sight bill on London, for £500 10*s*., cost in New York, exchange being at $4.87?

OPERATION

$$\begin{array}{r} 10\,s. = £\,.5 \\ 500.5 \\ \underline{4.87} \\ 35035 \\ 40040 \\ \underline{20020} \\ \$2437.435 \end{array}$$

SOLUTION. — Since 20 *s.* = £ 1, 10 *s.* = £ .5. If £ 1 is worth \$4.87, £ 500.5 are worth 500.5 × \$4.87 = \$2437.44.

2. How large a bill on London can be bought for \$1808.04, exchange being at \$4.88?

OPERATION

$$\begin{array}{l} \$4.88)\$1808.04(370 \\ \underline{1464} \\ 3440 \\ \underline{3416} \\ 244 \\ \underline{20} \\ 488)4880(10\,s. \\ \underline{488} \\ 0 \end{array}$$

SOLUTION. — Since £ 1 is worth \$4.88, as many pounds can be bought for \$1808.04 as \$4.88 is contained times in \$1808.04. It is contained 370$\frac{244}{488}$ times. Reduce £ $\frac{244}{488}$ to 10 *s.* by multiplying by 20. The bill will be for £ 370 10 *s.*

3. How much will a bill on London for £890 8 *s.* cost, exchange being at \$4.86? \$4327.34.

4. How large a bill on London can be bought for \$2130.12, exchange being at \$4.88? £436 10 *s.*

5. How much will a bill on Paris cost for 1290 francs, exchange being 5 fr. 15 centimes to \$1? \$250.49.

6. How large a bill on Paris can be bought for \$1657.60, exchange being at 5 fr. 16 centimes? 8553 fr. 22.

7. How much will a bill on Berlin cost for 12680 reichsmarks, exchange being \$.97 per 4 reichsmarks? \$3074.90.

8. How large a bill on Frankfort can be bought for \$1470, exchange being at \$.98? 6000 m.

INSURANCE

204. **Insurance companies** agree, for specified sums of money, to pay a certain amount to the person insured on the occurrence of a certain event.

The **policy** is the written contract given by the company to the person insured.

NOTE.—The persons insured are called **policy holders.** The companies are sometimes called **underwriters.**

The **premium** is the sum paid to the company for insurance.

Fire insurance is indemnity for a certain amount in case of loss by fire.

Marine insurance is indemnity for a certain amount in case of loss by the dangers of navigation.

Life insurance is an agreement to pay a specified sum at the death, or at a certain time in the life, of the insured.

FIRE AND MARINE INSURANCE

205. The **premium** in fire and marine insurance is a certain percentage of the amount insured (§ 170, Rule).

NOTE.—Insurance companies often insure property at only part of its value.

1. What is the cost of insuring a house worth \$3375, at $\frac{2}{3}$ of its value, the premium being $1\frac{1}{2}\%$?

SOLUTION.—$\frac{2}{3}$ of the value of the house is \$2250. The premium is $1\frac{1}{2}\%$ of \$2250, which is \$33.75.

OPERATION

3)\$3375	\$2250
\$1125	$.01\frac{1}{2}$
2	1125
\$2250	2250
	\$33.75

2. What is the cost of insuring a house worth \$5000, at $\frac{3}{4}$ of its value, the premium being $\frac{1}{2}$%? \$18.75.

3. A store is valued at \$12600, and the goods at \$14400; $\frac{5}{8}$ of the value of the store is insured at $\frac{3}{4}$%, and $\frac{1}{2}$ the value of the goods is insured at 2%. What is the total cost of insurance? \$207.

4. A man's dwelling, valued at \$5600, was burned; it had been insured, in a certain company, 20 years, for $\frac{3}{4}$ of its value, at $1\frac{1}{2}$%. How much did he receive from the company more than the sum total of the annual premiums? \$2940.

5. A man secures a policy of insurance, on his house, for \$3600, furniture for \$1600, and library \$800; the premium is $\frac{7}{8}$%. What is the cost of the insurance? \$52.50.

6. A hotel is insured, for $\frac{2}{3}$ of its value, at $1\frac{1}{2}$%; and the cost of insurance is \$150. At what sum is the hotel valued? \$15000.

7. The cost of insuring a house worth \$4500, for $\frac{4}{5}$ of its value, was \$31.50. What was the per cent of insurance? $\frac{7}{8}$%.

8. A farmer, with an insurance of \$1000 on his house, and \$1500 on his barn, pays an annual premium of \$12.50. What is the per cent of the premium? $\frac{1}{2}$%.

LIFE INSURANCE

206. Life insurance policies are of two principal kinds, (1) *life* policies; (2) *endowment* policies.

A life policy is payable at the death of the person insured.

An endowment policy is payable at a specified time, or at death if it occurs within this time.

NOTE.—In life insurance the premium is commonly a regular annual payment, dependent, in amount, upon the age of the individual when he effects his insurance. The tables of each company show the annual premium, at any age, for $1000 of insurance.

1. A man at the age of 40 insures his life for $5000; the company's annual premium on $1000, for a life policy at this age, is $31.30. If he dies at the age of 70, how much money will he have paid the company?

SOLUTION.—Since the annual premium on $1000 is $31.30, on $5000 it is 5 × $31.30 = $156.50; the amount paid, in 30 yr., will be 30 × $156.50 = $4695.

OPERATION

$$\begin{array}{r} \$31.30 \\ 5 \\ \hline \$156.50 \\ 30 \\ \hline \$4695.00 \end{array}$$

2. Mr. Harris, aged 35, takes out an endowment policy in a life insurance company for $10000, payable in 10 years; the cost of the annual premium on $1000, at his age, is $105.53. If he lives to receive the endowment, what will be the cost of the paid-up policy, without interest?

$10553.

3. At the age of 50, the cost of a life policy, payable annually, is $47.18 on $1000; the cost of an endowment policy, payable in 20 years, is $60.45 on $1000. At the end of 20 years, how much more will have been paid on a policy of $8000 by the endowment plan than by the life plan? $2123.20.

4. At the age of 21, a young man takes out a life policy for $5000, upon which the annual premium is $19.89 on $1000. If he lives to the age of 75, how much will it cost him to keep up his insurance? $5370.30.

EXCHANGE

200. A **draft**, or a **bill of exchange**, is a written order, from one person to another, to pay a certain amount of money at a specified time to the party named in the order.

NOTES. — 1. The person upon whom the bill is drawn is called the **drawee**; the person in whose favor it is drawn is called the **payee**.

2. When the draft is to be paid upon presentation it is called a **sight draft**; when it is to be paid at the end of a certain time it is called a **time draft**.

3. An order on a bank by a person who has deposited money there is called a **check**.

Exchange is the method of making payments in distant places by means of a draft, or a bill of exchange.

There are two sorts of exchange, *domestic* and *foreign*.

DOMESTIC EXCHANGE

Domestic exchange takes place between localities in the same country.

NOTE. — When the banks of, say, Chicago have not enough money deposited in New York to meet their drafts on New York, they must send on money. The price in Chicago of drafts on New York will then be at a **premium**. But when the banks of Chicago have a large surplus deposited in New York it is desirable for them to sell drafts on New York, and drafts in Chicago on New York will be at a **discount**.

The following is a common form of a domestic bill of exchange, which is usually termed a *draft*.

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To Charles Smith & Co., Bankers, New York.

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£500. CINCINNATI, O., May 1, 1902.

At sight of this Original of Exchange (Duplicate of the same tenor and date unpaid), pay to the order of Amos Carroll, Five hundred Pounds Sterling, value received, and charge to account of STANLEY BINGHAM.

To Baring Brothers, London.

A foreign bill of exchange is usually drawn in duplicate; the different copies, termed respectively the original and duplicate or *first* and *second* of exchange, are then sent by different mails, that miscarriage or delay may be avoided. When one is paid the other is void.

The **acceptance** of a bill of exchange is the agreement by the drawee to pay it when due.

NOTE.—A bill is accepted by the drawee's writing the word "accepted," with his name, across the face of the bill; the bill is then an **acceptance**.

201. To find the cost or face of a domestic bill of exchange (§ **170**, Rule):

NOTE.—Interest and premium are calculated on the face of the draft.

1. What is the cost of a sight draft on New York for \$1400, at $\frac{1}{2}\%$ premium? \$1407.

2. What is the cost of a sight draft on Boston for \$2580, at $\frac{1}{2}\%$ discount? \$2567.10.

3. What is the face of a sight draft on Wheeling, which cost \$375.87, at $\frac{1}{8}\%$ premium? \$375.40.

The valuation is made by an officer called an **assessor.**

NOTE.—This official makes out a list called an **assessment roll**; it contains the names of the persons to be taxed, and the valuation of their property.

209. To find the rate of taxation:

The rate of taxation is expressed as so many mills on each dollar of taxable property, or as such a per cent of it.

1. The property of a certain town is valued at \$1049905; there are 483 persons subject to poll tax. In a certain year the total taxes of the town are \$13323.36; the poll tax being \$1.50 for each person, what is the rate of taxation upon the property?

OPERATION

```
              $1.50
                483
                450
              1200
              600
            $724.50
          $13323.36
             724.50
$1049905)$12598.86($.012
          10499 05
           2099 810
           2099 810
```

SOLUTION.—The poll tax is 483 × \$1.50 = \$724.50. Therefore the property tax is \$13323.36 − \$724.50 = \$12598.86. Since the tax on \$1049905 is \$12598.86, the tax on \$1 is \$12598.86 ÷ \$1049905 = \$0.012, 12 mills or $1\frac{1}{5}$%.

Rule.—1. *Multiply the tax on each poll by the number of polls; the product is the poll tax.*

2. *From the total amount of tax subtract the poll tax; the remainder is the property tax.*

3. *Divide the property tax by the valuation; the quotient is the rate of taxation.*

NOTE.—Of course, where there is no specific poll tax, the total amount of the tax is to be divided immediately by the valuation.

2. A tax of \$2500 is assessed upon a certain district to build a schoolhouse. The property of the district is valued at \$618000, and there are 28 persons subject to poll tax. If the poll tax is \$1, what will be the rate of taxation? 4 mills on \$1, or $\frac{2}{5}$%.

3. Upon a valuation of \$2876475 the tax is \$18409.44. There being no poll tax, what is the rate? 6.4 mills on \$1.

4. The total valuation of property in a certain state for 1902 was \$421285359; the tax levied upon this valuation was \$656491.61. What was the rate to the hundredth of a mill? 1.56 mills on \$1.

210. To apportion the tax among the tax payers:

I. A tax of \$1373.64 is assessed upon a village, the property of which is valued at \$748500; 57 persons pay a poll tax of \$1.25 each; find the rate of taxation, and construct a tax table to \$9000.

TAX TABLE

Rate, 1.74 mills on \$1

PROP.	TAX	PROP.	TAX	PROP.	TAX	PROP.	TAX
\$1	\$0.002	\$10	\$0.017	\$100	\$0.174	\$1000	\$ 1.74
2	.003	20	.035	200	.348	2000	3.48
3	.005	30	.052	300	.522	3000	5.22
4	.007	40	.070	400	.696	4000	6.96
5	.009	50	.087	500	.870	5000	8.70
6	.010	60	.104	600	1.044	6000	10.44
7	.012	70	.122	700	1.218	7000	12.18
8	.014	80	.139	800	1.392	8000	13.92
9	.016	90	.157	900	1.566	9000	15.66

NOTE.—In order to facilitate the calculation of each person's tax, it is customary to construct such a table. It is not necessary to carry it out in any column farther than the nearest mill.

1. James Turner's property is valued at \$7851, and he pays poll tax for 2 persons. What is his tax?

OPERATION

\$ 7851
\$12.180
1.392
.087
.002
\$13.66
2.50
\$16.16

SOLUTION. — By the table the tax on \$7000 is \$12.18; on \$800, \$1.392; on \$50, \$0.087; and on \$1, \$0.002; then the tax on \$7851 is \$12.18 + \$1.392 + \$0.087 + \$0.002 = \$13.66; this is his property tax. The poll tax is 2 × \$1.25 = \$2.50. Then James Turner's tax is \$13.66 + \$2.50 = \$16.16.

EXPLANATION. — It is evident that the operation is equivalent to multiplying \$7851 by the rate, 1.74, and adding the poll tax.

2. John Brown's property is valued at \$2576, and he pays poll tax for 1 person. What is his tax? \$5.73.

3. Henry Adams' property is valued at \$9265, and he pays poll tax for 3 persons. What is his tax? \$19.87.

4. Amos Clarke's property is valued at \$4759, and he pays poll tax for 1 person. What is his tax? \$9.53.

5. Emily Wood's property is valued at \$8367. What is her tax? \$14.56.

II. The tax to be raised in a city is \$64375; its taxable property is valued at \$16869758. Find the rate of taxation to thousandths of a mill, and construct a tax table to \$90000. Rate, 3.816 mills on \$1.

1. William Mill's property is valued at \$56875. What is his tax? \$217.04.

2. Samuel Young's property is valued at \$27543. What is his tax? \$105.10.

3. Charles O'Neil's property is valued at \$83612. What is his tax? \$319.06.

4. Adolph Meyer's property is valued at $72968. What is his tax? $278.45.

5. Louis Ganot's property is valued at $69547. What is his tax? $265.39.

UNITED STATES INTERNAL REVENUE

211. The **United States revenue** arises wholly from indirect taxation; it consists of *internal revenue* and the revenue from *duties* or *customs*.

The **internal revenue** arises from the sale of public lands, from a tax upon certain manufactures, from the sale of postage stamps, revenue stamps, etc.

212. 1. If the public lands are disposed of at $1.25 per acre, how much will the government receive for a township containing 36 square miles? $28800.

2. Letter postage is 2¢ for each ounce, or fraction thereof. What is the necessary postage on a letter weighing $1\frac{1}{4}$ oz.? 4¢.

3. The postage on books is 1¢ for each 2 oz., or fraction thereof. What is the postage on a book weighing 1 lb. 5 oz.? 11¢.

4. The tax on beer is $1.60 per barrel of 31 gallons. What is the tax on 5 barrels? $8.

5. The tax on cigars per 1000 is $3. How much does this enhance the price of a single cigar? $\frac{3}{10}$¢.

DUTIES OR CUSTOMS

213. **Duties** or **customs** are taxes on goods imported from foreign countries.

Duties are of two kinds, *specific* and *ad valorem*.

A **specific** duty is levied upon the quantity of the goods.

NOTE. — Sometimes this duty is levied at a certain rate per dozen articles, sometimes by the measurement of the article, and sometimes by the weight.

An **ad valorem** duty is levied upon the cost of the goods.

NOTES. — 1. The cost of the goods is shown by the foreign invoice, and it is determined by appraisement at the custom house.

2. The duty is computed on the net weight and on the total cost of the article in the foreign country. The *dutiable value* upon which the duty is estimated is always the nearest exact number of dollars, pounds, etc. If the total cost is \$85.49, the dutiable value is \$85; if it is \$85.50 or \$85.51, etc., the dutiable value is \$86.00.

1. The gross weight of a hogshead of imported sugar is 1760 lb. Allowing $12\frac{1}{2}\%$ tare, what is the duty at .0095¢ per pound? \$14.63.

2. A manufacturer imported from Spain 40 bales of wool, of 400 lb. each, tare 5%. What was the duty at 11¢ per pound? \$1672.

3. A merchant imported a case of glassware; the cost of the ware in France was 365.15 francs, the case and charges were 57.15 francs, and the commission 5%. What was the duty at 60% ad valorem in U.S. money, reckoning the franc at \$0.193? \$51.60.

4. A merchant imported six cases of woolen cloth, net weight 1500 lb.; the cost in England was £500, cases and charges £8 4*s*. 6*d*., commission $2\frac{1}{2}\%$. What was the duty at 44¢ per pound and 50% ad valorem in U.S. money, estimating the pound at \$4.8665? \$1927.50.

RATIO AND PROPORTION

RATIO

214. **Ratio** is the relation of two numbers expressed by their quotient.

Thus, the ratio of 6 to 2 is $6 \div 2 = 3$; that is, 6 is 3 times 2.

The ratio of two numbers is indicated by writing the sign (:) between them.

Thus, 2:6 is read the ratio of 2 to 6.

The two numbers are styled the **terms** of the ratio.

The first term is called the **antecedent**, and the second term the **consequent**.

NOTE. — 6 : 2 is 3, a ratio between two abstract numbers. \$6 : \$2 is 3, a ratio between two concrete numbers. When the terms are concrete, both must be of *the same denomination.* To find 2 yd. : 2 ft., reduce the 2 yd. to ft.; 6 ft. : 2 ft. is 3. *The ratio is always an abstract number.*

215. Ratios are either *simple* or *compound.*

A **simple** ratio is a single ratio.

Thus, 2 : 6 is a simple ratio.

A **compound** ratio consists of two or more simple ratios.

Thus, $\left.\begin{matrix} 2:6 \\ 3:9 \end{matrix}\right\}$ is a compound ratio.

216. The antecedent of a ratio is a dividend or the numerator of a fraction; the consequent is a divisor or denominator; and the ratio is the quotient. Hence (§ **101**),

Principles. — I. ***A ratio is multiplied by multiplying the antecedent or by dividing the consequent.***

II. ***A ratio is divided by dividing the antecedent or multiplying the consequent.***

III. ***A ratio is not changed by multiplying or dividing both its terms by the same number.***

217. 1. What is the ratio of 6 to 3?

SOLUTION. — The ratio of 6 to 3 is 6 divided by 3, equal to 2.

OPERATION

$6 : 3$

$6 \div 3 = 2$

2. What is the ratio of $\frac{2}{3}$ to $\frac{3}{4}$?

1ST SOLUTION. — The ratio of $\frac{2}{3}$ to $\frac{3}{4}$ is $\frac{2}{3}$ divided by $\frac{3}{4}$, or $\frac{2}{3}$ multiplied by $\frac{4}{3}$, equal to $\frac{8}{9}$.

1ST OPERATION

$\frac{2}{3} : \frac{3}{4}$

$\frac{2}{3} \div \frac{3}{4}$

$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$

2D SOLUTION. — $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$. The ratio of $\frac{8}{12} : \frac{9}{12}$ is the same as $8 : 9$ or $\frac{8}{9}$.

2D OPERATION

$\frac{2}{3} = \frac{8}{12} \quad \frac{3}{4} = \frac{9}{12}$

$\frac{8}{12} : \frac{9}{12} = 8 : 9 = \frac{8}{9}$

3. If the ratio is 2 and the antecedent 6, what is the consequent?

SOLUTION. — Since 2 is a quotient and 6 a dividend, the divisor or consequent must be $6 \div 2$, or 3 (§ **216**).

OPERATION

$6 \div 2 = 3$

4. If the ratio is 2 and the consequent 3, what is the antecedent?

SOLUTION. — Since 2 is a quotient and 3 a divisor, the dividend or antecedent must be 2×3 or 6 (§ **216**).

OPERATION

$2 \times 3 = 6$

What is the ratio of:

5. 12 to 3?	4.	8. 56 to 8?	7.
6. 30 to 5?	6.	9. 5 to 10?	$\frac{1}{2}$.
7. 35 to 7?	5.	10. 7 to 21?	$\frac{1}{3}$.

11. 12 to 18? $\frac{2}{3}$.

12. 15 to 20? $\frac{3}{4}$.

13. 15 to 25? $\frac{3}{5}$.

14. 25 to 15? $1\frac{2}{3}$.

15. 36 to 28? $1\frac{2}{7}$.

16. 49 to 35? $1\frac{2}{5}$.

17. $\frac{1}{2}$ to $\frac{2}{3}$? $\frac{3}{4}$.

18. $\frac{1}{3}$ to $\frac{3}{4}$? $\frac{4}{9}$.

19. $\frac{1}{3}$ to $\frac{1}{2}$? $\frac{2}{3}$.

20. $\frac{1}{5}$ to $\frac{1}{3}$? $\frac{3}{5}$.

21. $1\frac{1}{2}$ to $\frac{3}{4}$? 2.

22. $3\frac{1}{2}$ to $2\frac{1}{4}$? $1\frac{5}{9}$.

23. $5\frac{5}{6}$ to $2\frac{1}{3}$? $2\frac{1}{2}$.

24. $6\frac{9}{10}$ to $4\frac{3}{5}$? $1\frac{1}{2}$.

25. \$18 to \$6? 3.

26. 54 days to 9 days? 6.

27. 96 men to 12 men? 8.

28. 221 bu. to 17 bu.? 13.

29. 1 ft. 9 in. to 3 in.? 7.

30. 5 yd. 1 ft. to 5 ft. 4 in.? 3.

31. 4 is the ratio and 13 the consequent. Find the antecedent. 52.

32. $\frac{5}{9}$ is the ratio and 27 the consequent. Find the antecedent. 15.

33. $\frac{7}{13}$ is the ratio of what number to 52? 28.

34. $2\frac{5}{8}$ is the ratio of what number to 24? 63

35. $4\frac{2}{5}$ is the ratio of what number to $1\frac{2}{3}$? $7\frac{1}{3}$.

36. 3 is the ratio of what to 75¢? \$2.25.

37. $\frac{7}{8}$ is the ratio of what to 4 lb. 8 oz.? 3 lb. 15 oz.

38. 2.6 is the ratio of what to \$4? \$10.40.

39. 4 is the ratio and 56 the antecedent. Find the consequent. 14.

40. $\frac{7}{10}$ is the ratio and 42 the antecedent. Find the consequent. 60.

41. $2\frac{3}{4}$ is the ratio of $23\frac{3}{8}$ to what number? $8\frac{1}{2}$.

42. $7\frac{5}{9}$ is the ratio of \$27.20 to what number? \$3.60.

SIMPLE PROPORTION

218. **Proportion** is an expression for the equality of two ratios.

Thus, 2 : 4 and 3 : 6 may form a proportion, for the ratio of each is $\frac{1}{2}$.

219. The proportion is indicated by writing : : between the ratios.

Thus, 2 : 4 : : 3 : 6 is read 2 is to 4 as 3 is to 6.

220. In a **simple** proportion both the ratios are simple.

Thus, 2 : 4 : : 3 : 6 is a simple proportion.

NOTE. — In a **compound** proportion one or both the ratios are compound. Thus, $\left.\begin{matrix}2:3\\3:4\end{matrix}\right\} :: \left.\begin{matrix}4:5\\5:8\end{matrix}\right\}$ is a compound proportion.

221. Every proportion consists of four *terms*.

The first and fourth terms of a proportion are called the **extremes**.

The second and third terms of a proportion are called the **means**.

The last term is said to be a *fourth proportional* to the other three taken in order.

Thus, in the proportion 2 : 4 : : 3 : 6, the extremes are 2 and 6; the means are 4 and 3; and 6 is a fourth proportional to 2, 4, and 3.

When three numbers form a proportion, the second number is said to be a **mean proportional** between the other two.

Thus, in the proportion 2 : 4 : : 4 : 8, 4 is a mean proportional between 2 and 8.

222. The operations of proportion depend upon the following principle:

Principle. — *In every proportion the product of the extremes is equal to the product of the means.*

Thus, in the proportion $2:4::3:6$, $2 \times 6 = 4 \times 3$.

1st. *If the product of the means is divided by one of the extremes, the quotient will be the other extreme.*

2d. *If the product of the extremes is divided by one of the means, the quotient will be the other mean.*

223. Given three terms of a proportion, to find the fourth:

1. $? : 6 :: 4 : 8$.

SOLUTION. — The product of the means 6 and 4 is 24; 24 divided by 8, one of the extremes, equals 3, the other extreme (§ **222**).

OPERATION

$$6 \times 4 = 24$$
$$24 \div 8 = 3$$

2. $4 : ? :: 3 : 6$.

SOLUTION. — The product of the extremes is 24; 24 divided by 3, one of the means, equals 8, the other mean (§ **222**).

OPERATION

$$4 \times 6 = 24$$
$$24 \div 3 = 8$$

Rule. — *Divide the product of the terms of the same name by the other given term.*

NOTE. — Indicate the operation and cancel whenever it is practicable (§ **91**).

3. $2 : 8 :: 6 : ?$ — 24.

4. $5 : 7 :: 10 : ?$ — 14.

5. $? : 8 :: 6 : 16$. — 3.

6. $5 : ? :: 6 : 12$. — 10.

7. $3 : 7 :: ? : 14$. — 6.

8. $7 : 14 :: 9 : ?$ — 18.

9. $\frac{2}{3} : \frac{3}{4} :: \frac{4}{5} : ?$ — $\frac{9}{10}$.

10. $\frac{3}{5} : ? :: \frac{9}{10} : 1\frac{1}{4}$. — $\frac{5}{6}$.

11. $? : 4\frac{2}{3} :: 7\frac{1}{2} : 10\frac{1}{2}$. — $3\frac{1}{3}$.

12. $4 : 6 :: 6 : ?$ — 9.

224. Proportion, when applied to the solution of concrete problems, has been styled *The Rule of Three*, because three terms are given to find the fourth.

225. The solution of a problem by proportion consists of two parts:

1st. *The statement;* that is, the proper arrangement of the numbers into a proportion.

2d. *The operation* of finding the required term.

NOTE.—In arranging the numbers in a proportion it is customary, though not necessary, to make the number or quantity required a *fourth proportional* to the other three; then the first three terms of the proportion always are given to find the fourth.

226. 1. If 2 yd. of cloth cost \$4, how much will 6 yd. cost?

SOLUTION.—Since the number required, or fourth term of the proportion, is dollars, the third term is \$4. Since the cost of 6 yd. will be greater than the cost of 2 yd., 6 yd. is the second term of the proportion and 2 yd. the first term. Dividing the product of 6 and 4 by 2 (§ **223**, Rule), the required term is \$12.

OPERATION

$2 : 6 :: 4 : ?$

$$\frac{\overset{3}{\cancel{6}} \times 4}{\cancel{2}} = 12$$

NOTE.—In this example the number of dollars is in a **direct ratio** to the number of yards; that is, the *greater* the number of yards the *greater* the number of dollars they will cost.

2. If 3 men can dig a cellar in 10 days, in how many days can 5 men dig it?

SOLUTION.—Since the number required, or fourth term of the proportion, is days, the third term is 10 da. Since 5 men will dig the cellar in a less number of days than 3 men, 3 men is the second term of the proportion and 5 men the first term. Dividing the product of 3 and 10 by 5 (§ **223**, Rule), the required term is 6 da.

OPERATION

$5 : 3 :: 10 : ?$

$$\frac{3 \times \overset{2}{\cancel{10}}}{\cancel{5}} = 6$$

NOTE.—In this example the number of days is in an *inverse ratio* to the number of men; that is, the *greater* the number of men the ***less*** the number of days in which they will dig the cellar.

Rule. — 1. *For the third term, write that number which is of the same denomination as the number required.*

2. *For the second term, write the* GREATER *of the two remaining numbers, when the fourth term is to be greater than the third; and the* LESS, *when the fourth term is to be less than the third.*

3. *Divide the product of the second and third terms by the first; the quotient will be the fourth term, or number required.*

3. If 3 men can dig a cellar in 12 days, how many men can dig it in 6 days? 6 men.

4. If 3 yd. cloth cost $8, what will be the cost of 6 yd.? $16.

5. If 5 chairs cost $30, how much will 3 of these chairs cost? $18.

6. If 3 lb. 12 oz. tea cost $3.50, what will be the cost of 11 lb. 4 oz.? $10.50.

7. If 2 lb. 8 oz. of tea cost $2, what quantity can you buy for $5? 6 lb. 4 oz.

8. If 4 hats cost $14, how much will 10 hats cost? $35.

9. If 3 caps cost 69¢, find the cost of 11 caps. $2.53.

10. If 4 yd. cloth cost $7, find the cost of 9 yd. $15.75.

11. If 8 yd. cloth cost $32, find the cost of 12 yd. $48.

12. If 12 yd. cloth cost $48, find the cost of 8 yd. $32.

13. If $32 purchases 8 yd. of cloth, how many yards will $48 buy? 12 yd.

14. If $48 purchases 12 yd. of cloth, how many yards can be bought for $32? 8 yd.

15. A man receives \$152 for 19 months' work. How much should he have for 4 months' work? \$32.

16. If 8 men perform a piece of work in 24 days, in what time can 12 men perform it? 16 da.

17. If 60 men perform a piece of work in 8 da., how many men will perform it in 2 days? 240 men.

18. If 15 oz. of pepper cost 25¢, how much will 6 lb. cost? \$1.60.

19. If 6 gal. of molasses cost \$2.70, how much will 26 gal. cost? \$11.70.

20. If 5 cwt. 85 lb. of sugar cost \$21.06, how much will 35 cwt. 25 lb. cost? \$126.90.

21. If $1\frac{1}{2}$ yd. of cloth cost \$2.50, what will be the cost of $1\frac{1}{8}$ yd.? \$$1.87\frac{1}{2}$.

22. If 90 bu. of oats supply 40 horses 6 da., how long will 450 bu. supply them? 30 da.

23. If 6 men build a wall in 15 da., how many men can build it in 5 da.? 18 men.

24. If 15 bu. of corn pay for 30 bu. of potatoes, how much corn can be had for 140 bu. potatoes? 70 bu.

25. If 3 cwt. 25 lb. of sugar cost \$11.30, what will be the cost of 16 cwt. 25 lb.? \$56.50.

26. If a perpendicular staff, 3 ft. long, casts a shadow 4 ft. 6 in., what is the height of a steeple whose shadow measures 180 ft.? 120 ft.

27. If a man performs a journey in 60 da., traveling 9 hr. each day, in how many days can he perform it by traveling 12 hr. a day? 45 da.

28. A merchant, failing, paid 60¢ on each dollar of his debts. He owed A \$2200, and B \$1800. How much did each receive? A \$1320. B \$1080.

29. A merchant, having failed, owes A \$800.30; B \$250; C \$375.10; D \$500; E \$115; his property, worth \$612.12, goes to his creditors. How much will this pay on the dollar? 30¢.

30. If the 4¢ loaf weighs 9 oz. when flour is \$5 a barrel, how much will it weigh when flour is \$4 a barrel? $11\frac{1}{4}$ oz.

31. I borrowed of A \$250 for 6 mo. How long should I lend \$300 to compensate A for the favor? 5 mo.

32. A starts on a journey, and travels 27 mi. a day; 7 da. later, B starts and travels the same road 36 mi. a day. In how many days will B overtake A? 21 da.

33. If William's services are worth $\$15\frac{2}{3}$ a month, when he labors 9 hr. a day, how much ought he to receive for $4\frac{2}{5}$ mo., when he labors 12 hr. a day? $\$91.91\frac{1}{9}$.

34. If 5 lb. of butter cost $\$\frac{5}{8}$, find the cost of $\frac{3}{4}$ lb. $\$\frac{3}{32}$.

35. If 6 yd. cloth cost $\$5\frac{3}{5}$, find the cost of $7\frac{3}{8}$ yd. $\$6\frac{53}{60}$.

36. If $\frac{1}{3}$ bu. wheat cost $\$\frac{1}{3}$, find the cost of $\frac{1}{2}$ bu. $\$\frac{1}{2}$.

37. If $1\frac{3}{4}$ yd. cloth cost $\$\frac{7}{24}$, find the cost of 2 yd. $\$\frac{1}{3}$.

38. If $\$29\frac{3}{4}$ buys $59\frac{1}{2}$ yd. of cloth, how much will $\$31\frac{1}{4}$ buy? $62\frac{1}{2}$ yd.

39. If .85 of a number equals 1.36, find .25 of the number. .40.

40. If 61.3 times a number equals 44.9942, what will 1.08 times the number equal? .79.

41. A wheel has 35 cogs. A smaller wheel working in it has 26 cogs. In how many revolutions of the larger wheel will the smaller gain 10 revolutions? $28\frac{8}{9}$.

42. If a grocer, instead of a true gallon, uses a measure deficient by 1 gill, what will be the *true* measure of 100 of these *false* gallons? $96\frac{7}{8}$ gal.

43. If the velocity of sound in air is 1142 feet per second, and the number of pulsations of a person 70 per minute, what is the distance of a cloud, if 20 pulsations are counted between the time of seeing a flash of lightning and hearing the thunder? 3 mi. 226 rd. 2 yd. $2\frac{1}{7}$ ft.

44. The length of a wall by a measuring line was 643 ft. 8 in., but the line was found to be 25 ft. 5.25 in. long, instead of 25 feet, its supposed length. What was the true length of the wall? 654 ft. 11.17 in.

PARTNERSHIP AND BANKRUPTCY

227. A **partnership** is an association of persons for the transaction of business. Such an association is called a *firm*, or *house*, and each member is a *partner*.

228. The **capital**, or **stock**, is the amount of money or property contributed by the partners.

The **assets** are the amounts due a firm, together with the property of all kinds belonging to it.

The **liabilities** of a firm are its *debts*.

The **net capital** is the *difference* between the assets and liabilities.

229. A **bankrupt** is one who has failed to pay his debts when due.

NOTE.— The assets of a bankrupt are usually placed in the hands of an **assignee**, whose duty it is to convert them into cash and divide the net proceeds among the creditors according to the amount of the claims.

1. A and B engage in trade. A's capital is $200; B's, $300; they gain $100. Find each partner's share.

SOLUTION. — The whole capital is \$200 + \$300 = \$500; of this A owns $\frac{200}{500} = \frac{2}{5}$, and B owns $\frac{300}{500} = \frac{3}{5}$ of the capital; hence, A's gain will be $\frac{2}{5}$ of \$100 = \$40, and B's gain will be $\frac{3}{5}$ of \$100 = \$60.

Or, SOLUTION. — The whole capital is \$200 + \$300 = \$500; then

\$500 : \$200 : : \$100 : \$40, A's share;

\$500 : \$300 : : \$100 : \$60, B's share.

Rule. — *Take such part of the whole gain or loss, as each partner's stock is part of the whole stock.*

Or, *As the whole stock is to each partner's stock, so is the whole gain or loss to each partner's gain or loss.*

NOTE. — This rule is applicable when it is required to divide a sum into parts having a given ratio to each other.

2. A and B form a partnership, with a capital of \$800. A's part is \$300; B's, \$500. They gain \$232. What is the share of each? A's, \$87; B's, \$145.

3. A's stock was \$70; B's, \$150; C's, \$80. They gained \$120. What was each man's share of it?
A's, \$28; B's, \$60; C's, \$32.

4. A, B, and C traded together. A put in \$200; B, \$400; C, \$600. They gained \$427.26. Find each man's share. A's, \$71.21; B's, \$142.42; C's, \$213.63.

5. Divide \$90 among 3 persons, so that the parts shall be to each other as 1, 3, and 5. \$10, \$30, and \$50.

6. Divide \$735.93 among 4 men, in the ratio of 2, 3, 5, and 7. \$86.58; \$129.87; \$216.45; \$303.03.

7. A man leaves an estate of \$22361 to be divided among 6 children, in the ratio of their ages, which are 3, 6, 9, 11, 13, and 17 yr. What are the shares?
\$1137; \$2274; \$3411; \$4169; \$4927; \$6443.

8. Divide \$692.23 into 3 parts, that shall be to each other as $\frac{1}{3}$, $\frac{3}{5}$, and $\frac{7}{8}$. \$127.60; \$229.68; \$334.95.

9. A man, failing, owes A \$175; B, \$500; C, \$600; D, \$210; E, \$42.50; F, \$20; G, \$10. His property is worth \$934.50. What will be each creditor's share?

A's, \$105; C's, \$360; E's, \$25.50;
B's, \$300; D's, \$126; F's, \$12.00; G's, \$6.

10. A man owes A \$234; B, \$175; C, \$326. His property is worth \$492.45. How much can he pay on \$1; and how much will each creditor get?

67¢ on \$1; A, \$156.78; B, \$117.25; C, \$218.42.

11. Mr. Smith failed in business, owing \$37000. His assignee sold the stock for \$25000, and charged \$4650 for expenses. How much did he pay on the dollar? 55¢.

230. 1. A and B built a wall for \$82; A had 4 men at work 5 days, and B, 3 men 7 days. How should they divide the money?

SOLUTION.—The work of 4 men 5 da. equals the work of 5×4, or 20 men 1 da.; and the work of 3 men 7 da. equals the work of 7×3, or 21 men 1 da.; it is therefore required to divide \$82 into two parts having the same ratio to each other as 20 to 21; hence, A's part is $\frac{20}{41}$ of \$82 = \$40; B's part is $\frac{21}{41}$ of \$82 = \$42.

2. A put in trade \$50 for 4 mo.; B, \$60 for 5 mo.; they gained \$24. What was each man's share?

SOLUTION.—\$50 for 4 mo. equals $4 \times$ \$50 = \$200 for 1 mo.; and \$60 for 5 mo. equals $5 \times$ \$60 = \$300 for 1 mo. Hence, divide \$24 into two parts having the same ratio as 200 to 300, or 2 to 3. This gives A $\frac{2}{5}$ of \$24 = \$9.60, and B $\frac{3}{5}$ of \$24 = \$14.40.

Rule. — *Multiply each partner's stock by the time it was employed; then take such part of the gain or loss as each partner's product is part of the sum of all the products.*

3. A and B hire a pasture for \$54. A pastures 23 horses 27 da.; B, 21 horses 39 da. How much should each pay? A, \$23.28$\frac{3}{4}$; B, \$30.71$\frac{1}{4}$.

4. A put in \$300 for 5 mo.; B, \$400 for 8 mo.; C, \$500 for 3 mo.; they lost \$100. Find each one's loss. A's, \$24.19$\frac{11}{31}$; B's, \$51.61$\frac{9}{31}$; C's, \$24.19$\frac{11}{31}$.

5. A, B, and C hire a pasture for \$18.12. A pastures 6 cows 30 da.; B, 5 cows 40 da.; C, 8 cows 28 da. How much shall each pay? A, \$5.40; B, \$6; C, \$6.72.

6. Two men formed a partnership for 16 mo. A put in, at first, \$300, and, at the end of 8 mo., \$100 more; B put in, at first, \$600, but at the end of 10 mo., drew out \$300; they gained \$442.20. Find each man's share. A's, \$184.80; B's, \$257.40.

7. A and B are partners. A put in \$800 for 12 mo., and B, \$500. What sum must B put in at the end of 7 mo. to entitle him to half the year's profits? \$720.

8. A, B, and C form a partnership. A puts in \$2800 for 10 months, B \$3200 for 1 year, and C \$4000 for 8 months. They gain \$1230. What is each one's share of the gain? A's \$350, B's \$480, C's \$400.

9. A, B, and C enter into partnership. A puts in \$11,000 for 3 months, B puts in \$9900 for 5 months, C puts in \$7700 for 5 months. They gain \$5500. What is each partner's share of the gain?
A's \$1500, B's \$2250, C's \$1750.

INVOLUTION

231. **Involution** is the multiplication of a number by itself one or more times.

232. A **power** is the product obtained by involution.

The **first power** is the number itself.

The **second power**, or **square**, is the product obtained by taking the number twice as a factor.

Thus, $2 \times 2 = 4$, is the second power or square of 2.

The **third power**, or **cube**, is the product obtained by taking the number three times as a factor.

Thus, $2 \times 2 \times 2 = 8$ is the third power or cube of 2.

NOTE.—The second power is called the square, because the area of a square is the product of two equal factors (§ **68**). The third power is called the cube, because the solid contents of a cube is the product of three equal factors (§ **70**).

The higher powers of a number are denominated respectively the *fourth power*, *fifth power*, *sixth power*, etc.

Thus, $2 \times 2 \times 2 \times 2 = 16$ is the fourth power of 2; $2 \times 2 \times 2 \times 2 \times 2 = 32$, is the fifth power of 2; $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$ is the sixth power of 2, etc.

233. The **exponent** is a number denoting the power to which the given number is to be raised.

Thus, in 3^2, read 3 *square*, the 2 denotes the square of 3; hence, $3^2 = 9$. In 5^3, read 5 *cube*, the 3 denotes the cube of 5; hence, $5^3 = 125$. 7^4 is read 7 *fourth power*; 9^5, 9 *fifth power*, etc.

234. To raise a number to any power:

1. Find the cube of 75.

OPERATION

75	5625
75	75
375	28125
525	39375
5625	421875

SOLUTION. — 75 multiplied by 75 is 5625; this is the square of 75. 5625 multiplied by 75 is 421875; this is the cube of 75.

Rule. — *Obtain a product in which the number is taken as a factor as many times as there are units in the exponent of the power.*

2. Find the square of 65. 4225.
3. Find the cube of 25. 15625.
4. Find the fourth power of 12. 20736.
5. Find the fifth power of 10. 100000.
6. Find the sixth power of 9. 531441.
7. Find the eighth power of 2. 256.
8. Find the square of $\frac{2}{3}$. $\frac{4}{9}$.
9. Find the cube of $\frac{3}{4}$. $\frac{27}{64}$.
10. Find the fourth power of $\frac{4}{5}$. $\frac{256}{625}$.
11. Find the fifth power of $\frac{2}{3}$. $\frac{32}{243}$.
12. Find the square of $16\frac{1}{2}$. $272\frac{1}{4}$.
13. Find the cube of $12\frac{1}{2}$. $1953\frac{1}{8}$.
14. Find the fourth power of .25. .00390625.
15. 14^3 = what? 2744.
16. 19^4 = what? 130321.
17. $(2\frac{1}{3})^5$ = what? $69\frac{40}{243}$.

EVOLUTION

235. Evolution is the process of resolving a number into two or more equal factors.

A **root** of a number is one of the two or more equal factors.

The **square root** of a number is one of two equal factors.

Thus, 3 is the square root of 9; for $9 = 3 \times 3$.

The **cube root** of a number is one of three equal factors.

Thus, 3 is the cube root of 27; for $27 = 3 \times 3 \times 3$.

The higher roots of a number are denominated respectively the *fourth root*, *fifth root*, etc.

Thus, 3 is the fourth root of 81; for $81 = 3 \times 3 \times 3 \times 3$. 3 is the fifth root of 243; for $243 = 3 \times 3 \times 3 \times 3 \times 3$.

The **radical sign** $\sqrt{}$ placed before a number shows that its root is to be extracted.

The **index** is a number placed above the radical sign to show the number of the root.

NOTE.—It is customary, however, to omit 2, the index of the square root.

Thus, $\sqrt{25}$ is read the square root of 25; hence, $\sqrt{25} = 5$. $\sqrt[3]{27}$ is read the cube root of 27; hence, $\sqrt[3]{27} = 3$. $\sqrt[4]{16}$ is read the fourth root of 16; hence, $\sqrt[4]{16} = 2$.

A **perfect power** is one whose root can be obtained exactly.

Thus, 25 and $\frac{16}{25}$ are perfect squares: 27 and $\frac{8}{27}$ are perfect cubes; 16 and $\frac{1}{16}$ are perfect fourth powers.

The squares and cubes of the first ten numbers are exhibited in the following table:

Numbers	1	2	3	4	5	6	7	8	9	10
Squares	1	4	9	16	25	36	49	64	81	100
Cubes	1	8	27	64	125	216	343	512	729	1000

NOTE. — The numbers in the first horizontal line are the square roots of the corresponding numbers in the second line, and the cube roots of those in the third line.

An **imperfect power** is one whose root can be obtained only approximately. Thus, $\sqrt{2} = 1.41421+$.

SQUARE ROOT

236. To find the number of figures in the square root:

1. The square root of 1 is 1, and the square root of 100 is 10 (§ **235**, Table); between 1 and 100 are all numbers consisting of one or two figures, and between 1 and 10 are all numbers consisting of one figure. Therefore,

When a number consists of one or two figures, its square root consists of one figure.

2. The square root of 100 is 10, and the square root of 10000 is 100; between 100 and 10000 are all numbers consisting of three or four figures, and between 10 and 100 are all numbers consisting of two figures. Therefore,

When a number consists of three or four figures, its square root consists of two figures.

3. In like manner it may be shown that,

When a number consists of five or six figures, its square root consists of three figures.

Principles. — I. *If a number is pointed off into periods of two figures each, the number of periods will be the same as the number of figures in the square root.*

II. *The square of the units will be found in the first period, the square of the tens in the second period, the square of the hundreds in the third period, etc.*

1. Point off 368425. $3\dot{6}8\dot{4}2\dot{5}.$

2. Point off 6.843256. $\dot{6}.8\dot{4}3\dot{2}5\dot{6}.$

3. Point off 83751.42963. $\dot{8}3\dot{7}5\dot{1}.4\dot{2}9\dot{6}3\dot{0}.$

Rule. —*Place a point over the order units, and then over every second order from units to the left and to the right.*

NOTES.—1. The first period on the left of the integral part of the number will often contain but a single figure.

2. When the first period on the right of the decimal part contains but a single figure, a cipher must be annexed to complete the period.

4. Point off 864326; 4.758462; 7584.3769.

5. Point off 97285.46138; 75300; .046827; .0625; .625.

237. To find the square of a number in terms of its tens and units:

1. Find the square of 16 in terms of its tens and units.

SOLUTION. — $16 = 10 + 6$. Multiplying $10 + 6$ by 6 and indicating the operation, we have (10×6) and (6×6) or 6^2. Multiplying $10 + 6$ by 10, we have (10×10) or 10^2 and (10×6), which write under the first (10×6). The sum is $10^2 + 2(10 \times 6) + 6^2$, which equals 256. 10^2 is the square of the tens; $2(10 \times 6)$ is 2 times the tens by the units; 6^2 is the square of the units. Hence,

OPERATION

$$
\begin{array}{rcr}
16 & = & 10+6 \\
16 & = & 10+6 \\
\hline
96 & = & (10 \times 6) + 6^2 \\
16 & & 10^2 + (10 \times 6) \\
\hline
256 & = & 10^2 + 2(10 \times 6) + 6^2
\end{array}
$$

The square of a number consisting of tens and units equals the square of the tens plus twice the product of the tens by the units plus the square of the units.

NOTES. — 1. $2(10 \times 6)$ is the same as $2 \times (10 \times 6)$. See § **49**.

2. When a number is separated into *any* two parts its square always equals the square of the first part plus twice the product of the first part by the second plus the square of the second part. Thus, $156 = 15$ tens + 6 units; and $156^2 = (15 \text{ tens})^2 + 2(15 \text{ tens} \times 6 \text{ units}) + (6 \text{ units})^2$, or 24336.

If we express "tens" by "t," and units by "u," we have the formula $(t + u)^2 = t^2 + 2\,tu + u^2$.

238. To extract the square root of a number:

1. Extract the square root of 256.

OPERATION

$$\begin{array}{r|l} & \dot{2}5\dot{6}(16 \\ & \underline{1} \\ 26) & 156 \\ & \underline{156} \end{array}$$

SOLUTION. — Point off 256 into periods of two figures each by placing a point over 6 and 2 (§ **236**, Rule).

The largest square in 2 (§ **235**, Table) is 1; its root is 1; place the root 1 on the right and subtract the square 1 from 2; the remainder is 1, to which bring down the next period, 56.

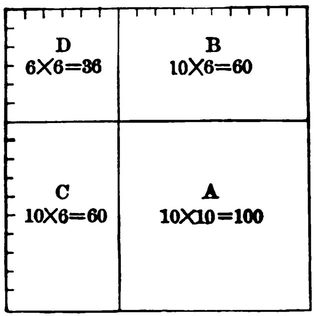
Double the root 1 and place the result 2 on the left of 156 for a trial divisor. Find how many times 2 is contained in 15 (making allowance for subsequent increase of the trial divisor); the result is 6; place 6 in the root on the right of 1 and also on the right of 2, the trial divisor; then 26 is the complete divisor. Multiply 26 by 6 and subtract the product 156 from 156; the remainder is 0. Therefore, 256 is a perfect square, and its square root is 16.

Arithmetical Explanation

The greatest square in 256 is 100, that is, 1 hundred or t^2 of the formula $(t^2 + 2\,tu + u^2)$, and its square root is 1 (ten). Subtracting 1 hundred from the whole square, 256 or $t^2 + 2\,tu + u^2$, the remainder 156 must represent $2\,tu + u^2$, that is, twice the tens by the units plus the square of the units. Since you do not know the units, take as a

trial divisor, twice the tens or 20, which is contained in 156 about 6 times. The complete divisor is 26 (that is, 2 tens + 6 units), which multiplied by 6, the units' figure, gives 156, which is twice the tens by the units plus the units squared ($2\,tu + u^2$). Since you have now exhausted the whole of 256 or $t^2 + 2\,tu + u^2$, 16 is the square root of 256.

Geometrical Explanation



After finding that the square root of the given number will contain two places of figures (tens and units), and that the figure in tens' place is 1 (ten), form a square figure (A) 10 in. on each side, which contains (§ **67**) 100 sq. in.; taking this sum from the whole number of squares, 156 sq. in. remain, which correspond to the number 156, left after subtracting on p. 285.

It is obvious that to increase the figure A, and at the same time preserve it a square, both length and breadth must be *increased equally*; and, since each side is 10 in. long, it will take *twice* 10, that is, 20 in., to encompass two sides of the square A. For this reason, 10 is doubled in the numerical operation.

Now determine the breadth of the addition to be made to each side of the square A. After increasing each side equally, it will require a small square, D, of the *same breadth* as each of the figures B and C, to complete the entire square; hence, the superficial contents of B, C, and D, must be equal to the remainder, 156. Now their contents are obtained by multiplying their length by their breadth.

Then the figure in the units' place — that is, the breadth of B and C — must be found by *trial*, and it will be somewhat less than the number of times the length of B and C (20) is contained in the remainder (156). 20 is contained in 156 more than 7 times; let us try 7; 7 added to 20 makes 27 for the whole length of B, C, and D, and this, multiplied by 7, gives 189 for their superficial contents; this being more than 156, the breadth (7) was taken too great. Next,

try 6 for the length and breadth of D; adding 6 to 20 gives 26 for the length of B, C, and D; multiplying 26 by the breadth (6) gives 156 for the superficial contents of B, C, and D.

Hence, the square root of 256 is 16; or, when 256 sq. in. are arranged in the form of a square, each side is 16 inches.

2. Extract the square root of 758.436.

OPERATION

758.4360(27.53 +
4
47)358
329
545)2943
2725
5503)21860
16509
5351

SOLUTION.—Point off 758.436 into periods of two figures each by placing a point over 8 and then over 7 to the left, and 3 and 0 to the right (§ **236**, Rule). Then find the figures of the root as in Ex. 1. The last remainder is 5351. Therefore, 758.436 is an imperfect square, and its square root is 27.53 +.

NOTE.—By bringing down one or more periods of decimal ciphers, the operation might be continued to any required number of decimal places in the root.

3. Extract the square root of $\frac{256}{625}$.

SOLUTION.—The square root of the numerator 256 is 16, and the square root of the denominator 625 is 25 (Ex. 1); therefore, the square root of $\frac{256}{625}$ is $\frac{16}{25}$.

4. Extract the root of $\frac{3}{8}$.

SOLUTION.—$\frac{3}{8}$ reduced to a decimal is .375. The square root of .375, to five decimal places, is .61237 (Ex. 2); therefore, the square root of $\frac{3}{8}$ is .61237 +.

Rule.—1. *Point off the given number into periods of two figures each.*

2. *Find the greatest square in the first period on the left; place its root on the right, like a quotient in division; subtract the square from the period, and to the remainder bring down the next period for a dividend.*

3. *Double the root found, and place it on the left of the dividend for a trial divisor. Find how many times the trial divisor is contained in the dividend, exclusive of the right-hand figure; place the quotient in the root, and also on the right of the trial divisor.*

4. *Multiply the complete divisor by the last figure of the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

5. *Double the whole root found, for a new trial divisor, and continue the operation in the same manner until all the periods are brought down.*

NOTES.—1. When the number is an imperfect square, the operation may be continued to any required number of decimal places in the root by bringing down periods of decimal ciphers (Ex. 2).

2. To extract the square root of a common fraction: (1) when both terms are perfect squares, extract the square root of the numerator and then of the denominator (Ex. 3); (2) when both terms are not perfect squares, reduce the fraction to a decimal and extract the square root of the decimal (Ex. 4).

Extract the square root of:

5.	529.	23.	**17.**	915.0625.	30.25.
6.	625.	25.	**18.**	.0196.	.14.
7.	6561.	81.	**19.**	1.008016.	1.004.
8.	56644.	238.	**20.**	.00822649.	.0907.
9.	390625.	625.	**21.**	$\frac{25}{729}$.	$\frac{5}{27}$.
10.	1679616.	1296.	**22.**	$\frac{847}{1188}$.	$\frac{11}{13}$.
11.	5764801.	2401.	**23.**	$30\frac{1}{4}$.	$5\frac{1}{2}$.
12.	43046721.	6561.	**24.**	10.	3.16227+.
13.	987656329.	31427.	**25.**	2.	1.41421+.
14.	289442169.	17013.	**26.**	$\frac{2}{3}$.	.81649+.
15.	234.09.	15.3.	**27.**	$6\frac{2}{5}$.	2.5298+ .
16.	145.2025.	12.05.	**28.**	$384\frac{4}{7}$.	19.61049+.

239. To extract the square root of a perfect square by factoring:

1. Extract the square root of 441.

SOLUTION.—$441 = 3 \times 3 \times 7 \times 7$; hence, $\sqrt{441} = 3 \times 7 = 21$.

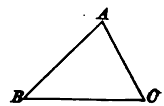
Rule.—*Resolve the number into its prime factors, and find the product of one of each two equal factors.*

Extract the square root of:

2. 16.	4.	6. 400.	20.
3. 36.	6.	7. 1764.	42.
4. 100.	10.	8. 5184.	72.
5. 225.	15.	9. 3025.	55.

240. Given two of the sides of a right-angled triangle to find the third side:

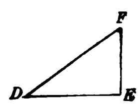
A **triangle** is a plane figure bounded by three straight lines, called its *sides*.



Thus, ABC is a triangle; its sides are AB, BC, and AC.

When one of the sides is perpendicular to another, they form a right angle, and the triangle is called a **right-angled** triangle.

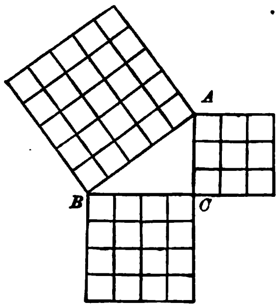
Thus, the triangle DEF is a right-angled triangle because the side FE is perpendicular to the side DE.



The side opposite the right angle is called the **hypotenuse**; the other two sides, the **base** and the **perpendicular**.

Thus, in DEF, DF is the hypotenuse, DE the base, and EF the perpendicular.

Proposition. — *The square described on the hypotenuse of a right-angled triangle is equal to the sum of the squares described on the other two sides.*



Draw a right-angled triangle, ABC, with the side BC 4 in., and the side AC 3 in.; then, the side AB will be 5 in. Describe a square on each side of the triangle, and divide each square into smaller squares of 1 in. to the side. The square described on AB will contain 25 square inches, and the two squares described on BC and AC will contain $16 + 9 = 25$ square inches.

I. To find the hypotenuse:

Rule. — *To the square of the base add the square of the perpendicular, and extract the square root of the sum.*

II. To find the base or the perpendicular:

Rule. — *From the square of the hypotenuse subtract the square of the other given side, and extract the square root of the difference.*

1. The base and perpendicular of a right-angled triangle are 30 and 40. What is the hypotenuse? 50.

2. The hypotenuse of a right-angled triangle is 100, and the base 60. What is the perpendicular? 80.

3. A castle 45 yd. high is surrounded by a ditch 60 yd. wide. What length of rope will reach from the outside of the ditch to the top of the castle? 75 yd.

4. A ladder 60 ft. long reaches a window 37 ft. from the ground on one side of the street, and, without moving it at the foot, will reach one 23 ft. high on the other side. Find the width of the street. 102.65 ft.

5. A tree 140 ft. high is in the center of a circular island 100 ft. in diameter; a line 600 ft. long reaches from the top of the tree to the further shore. What is the breadth of the stream, the land on each side being of the same level? 533.43 + ft.

6. A room is 20 ft. long, 16 ft. wide, and 12 ft. high. What is the distance from one of the lower corners to the opposite upper corner? 28.28 + ft.

241. Given the area of a square to find its side (§ 67):

Rule. — *Extract the square root of the area.*

1. The area of a square field is 6241 sq. rd. What is the length of one side? 79 rd.

2. The surface of a square table contains 8 sq. ft. 4 sq. in. What is the length of one side? 2 ft. 10 in.

3. The area of a circle is 4096 sq. yd. What is the side of a square of equal area? 64 yd.

4. A square field measures 4 rd. on each side. What is the length of the side of a square field which contains 9 times as many square rods? 12 rd.

5. What is the length of one side of a square lot containing 1 acre? 208.71 + ft.

CUBE ROOT

242. To find the number of figures in the cube root:

1. The cube root of 1 is 1, and the cube root of 1000 is 10 (§ 235, Table); between 1 and 1000 are all numbers consisting of one, two, or three figures, and between 1 and 10 are all numbers consisting of one figure. Therefore,

When a number consists of one, two, or three figures, its cube root consists of one figure.

2. The cube root of 1000 is 10, and the cube root of 1000000 is 100; between 1000 and 1000000 are all numbers consisting of four, five, or six figures, and between 10 and 100 are all numbers consisting of two figures. Therefore,

When a number consists of four, five, or six figures, its cube root consists of two figures.

3. In like manner it may be shown that,

When a number consists of seven, eight, or nine figures, its cube root consists of three figures.

And so on. Therefore,

Principles. — I. *If a number is pointed off into periods of three figures each, the number of periods will be the same as the number of figures in the cube root.*

II. *The cube of the units will be found in the first period, the cube of the tens in the second period, the cube of the hundreds in the third period, etc.*

1. Point off 876453921. $87\dot{6}45\dot{3}92\dot{1}$.

2. Point off 7.356849227. $\dot{7}.35\dot{6}84\dot{9}22\dot{7}$.

3. Point off 37683.5624. $3\dot{7}68\dot{3}.56\dot{2}40\dot{0}$.

Rule. — *Place a point over the order units, and then over every third order from units to the left and to the right.*

Notes. — 1. The first period on the left of the integral part of the number will often contain but one or two figures.

2. When the first period on the right of the decimal part contains but one or two figures, ciphers must be annexed to complete the period.

4. Point off 138975462; 3.561325482; 684536.256403.

5. Point off 2756.56843; 98451.3276; .856375; .0064.

243. To find the cube of a number in terms of its tens and units:

1. Find the cube of 24 in terms of its tens and units.

OPERATION

$$\begin{array}{rl}
24^2 = & (20+4)^2 = \\
576 = & 20^2 + 2(20 \times 4) + 4^2 \\
\underline{24} = & \underline{\qquad\qquad 20 + 4} \\
2304 = & (20^2 \times 4) + 2(20 \times 4^2) + 4^3 \\
\underline{1152} = & \underline{20^3 + 2(20^2 \times 4) + (20 \times 4^2)} \\
13824 = & 20^3 + 3(20^2 \times 4) + 3(20 \times 4^2) + 4^3
\end{array}$$

Solution. — The cube of $24 = 24 \times 24 \times 24$ or $24^2 \times 24$. The square of $20 + 4 = 20^2 + 2(20 \times 4) + 4^2$ (§ **237**). Multiplying this first by 4 and then by 20 and adding, the result is $20^3 + 3(20^2 \times 4) + 3(20 \times 4^2) + 4^3$, which equals 13824. 20^3 is the cube of the tens; $3(20^2 \times 4)$ is 3 times the square of the tens by the units; $3(20 \times 4^2)$ is 3 times the tens by the square of the units; 4^3 is the cube of the units.

The cube of a number consisting of tens and units equals the cube of the tens plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units plus the cube of the units.

If we express tens by t and units by u, we have the formula: $(t+u)^3 = t^3 + 3t^2u + 3tu^2 + u^3$.

244. To extract the cube root of a number:

1. Extract the cube root of 13824.

Solution. — Point off 13824 into periods of three figures each by placing a point over 4 and 3.

The largest cube in 13 is 8; its root is 2; place the root 2 on the right, and subtract the cube 8 from 13; the remainder is 5, to which bring down the next period 824.

Square the root 2 (tens) or 20 and multiply it by 3, the result,

OPERATION

$$\begin{array}{rr|l}
 & 1\dot{3}82\dot{4}(24 & \\
 & 8 & \\
3 \times 20^2 = & 1200 & 5824 \\
3 \times (20 \times 4) = & 240 & \\
4 \times 4 = & \underline{16} & \\
 & 1456 & \underline{5824}
\end{array}$$

1200, is the trial divisor. Find how many times 1200 is contained in 5824; the result is 4; place 4 in the root on the right of 2.

Multiply 3 times 2 tens by 4 and square 4; add the products 240 and 16 to 1200; the sum 1456 is the complete divisor. Multiply 1456 by 4, and subtract the product 5824 from 5824; the remainder is 0. Therefore, 13824 is a perfect cube, and its cube root is 24.

Arithmetical Explanation

The greatest cube in 13824 is 8000, that is, 8 thousand or t^3 of the formula (§ **243**), and its cube root is 2 (tens). Subtracting 8 thousand from the whole cube 13824, or $t^3 + 3\,t^2u + 3\,tu^2 + u^3$, the remainder 5824 must represent $3\,t^2u + 3\,tu^2 + u^3$, that is 3 times the product of the tens squared by the units, plus 3 times the product of the tens by the units squared, plus the units cubed. Since you do not know the units, take as a trial divisor 3 times the square of the 2 tens. The result 1200 ($3\,t^2$) is contained in 5824 about 4 times. Since $3\,t^2u + 3\,tu^2 + u^3 = u(3\,t^2 + 3\,tu + u^2)$, before multiplying the trial divisor 1200 ($3\,t^2$) by u, the units, we must add to it $3\,tu + u^2 \cdot 3\,tu = 3 \times 2$ tens $\times 4$ or $3 \times (20 \times 4) = 240$; $u^2 = 4 \times 4 = 16$. The complete divisor is $1200 + 240 + 16$ or 1456; which multiplied by 4, the units' figure, gives 5824. Since you have now exhausted the whole of 13824, or $t^3 + 3\,t^2u + 3\,tu^2 + u^3$, 24 is the cube root of 13824.

Geometrical Explanation

After finding that the cube root of the given number will contain two places of figures (tens and units), and that the figure in the tens' place is 2, form a cube, A, Fig. 1, 20 (that is, 2 tens) in. long, 20 in. wide, and 20 in. high; this cube will contain $20 \times 20 \times 20 = 8000$ cu. in.; take this sum from the whole number of cu. in., and 5824 cu. in. are left.

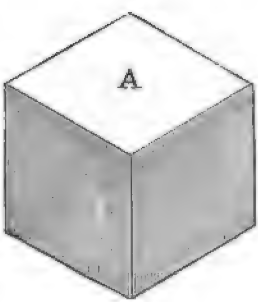


FIG. 1.

It is obvious that to increase the figure A, and at the same time preserve it a cube, the length, breadth, and height must each receive an equal addition. Then, since each side is 20 in. long, square 20, which gives $20 \times 20 = 400$, for the number of square inches in each face of the cube; and since an addition is to be

made to three sides, multiply the 400 by 3, which gives 1200 for the number of square inches in the 3 sides. This 1200 is called the **trial divisor**, because, by means of it, the *thickness* of the additions is determined.

By examining Fig. 2 it will be seen that, after increasing each of the three sides equally, there will be required 3 oblong solids, *C*, *C*, *C*, of the same length as each of the sides, and whose thickness and height are each the same as the additional thickness; and also a cube, *D*, whose length, breadth, and height are each the same as the additional thickness. Hence, the solid contents of the first three rectangular solids, the three oblong solids, and the small cube, must together be equal to the remainder (5824).

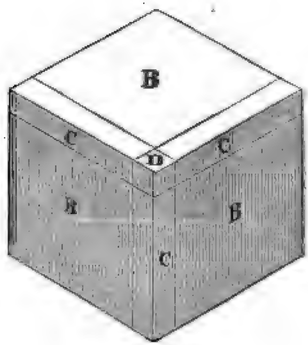


FIG. 2.

Now find the thickness of the additions. It will always be something less than the number of times the *trial divisor* (1200) is contained in the dividend (5824). By trial, we find 1200 is contained 4 times in 5824; proceed to find the contents of the different solids. The solid contents of the first three additions, *B*, *B*, *B*, are found by multiplying the number of square inches in the face by the thickness (§ **70**); there are 400 sq. in. in one face of each, and $400 \times 3 = 1200$ sq. in. in one face of the three; multiplying this by 4 (the thickness) gives 4800 cu. in. for their contents. The solid contents of the three oblong solids, *C*, *C*, *C*, are found by multiplying the number of square inches in the face by the thickness. There are $20 \times 4 = 80$ sq. in. in one face of each, and $80 \times 3 = 240$ sq. in. in one face of the three; multiplying by 4 (the thickness) gives 960 cu. in. for their contents. Lastly, find the contents of the small cube, *D*, by multiplying together its length, breadth, and thickness; this gives $4 \times 4 \times 4 = 64$ cu. in.

If the solid contents of the several additions are added together, as in the margin, their sum, 5824 cu. in., will be the number of cubic inches remaining after forming the first cube, *A*. Hence, when 13824 cu. in. are arranged in the form of a cube, each side is 24 in.; that is, the cube root of 13824 is 24.

ADDITIONS

$B, B, B =$	4800 cu. in.
$C, C, C =$	960 cu. in.
$D =$	64 cu. in.
Sum,	5824 cu. in.

2. Extract the cube root of 413.5147.

OPERATION

$$
\begin{array}{lcr|l}
 & & & 41\dot{3}.51\dot{4}70\dot{0}(7.45+ \\
 & & & 343 \\ \cline{4-4}
3 \times 70^2 & = & 14700 & 70514 \\
3 \times (70 \times 4) & = & 840 & \\
4 \times 4 & = & \underline{16} & \\
 & & 15556 & \underline{62224} \\
3 \times 740^2 & = & 1642800 & \quad 8290700 \\
3 \times (740 \times 5) & = & 11100 & \\
5 \times 5 & = & \underline{25} & \\
 & & 1653925 & \quad \underline{8269625} \\
 & & & \qquad 21075
\end{array}
$$

Solution. — Point off 413.5147 into periods of three figures each by placing a point over 3, and then over 4 and 0 to the right (§ **242**, Rule). Then find the figures of the root as in Ex. 1. The last remainder is 21075. Therefore, 413.5147 is an imperfect cube, and its cube root is 7.45+.

Note. — By bringing down one or more periods of decimal ciphers the operation might be continued to any required number of decimal places in the root.

3. Extract the cube root of $\frac{2197}{13824}$.

Solution. — The cube root of the numerator 2197 is 13 and the cube root of the denominator 13824 is 24 (Ex. 1); therefore, the cube root of $\frac{2197}{13824}$ is $\frac{13}{24}$.

4. Extract the cube root of $\frac{4}{5}$.

Solution. — $\frac{4}{5}$ reduced to a decimal is .8. The cube root of .8 to three decimal places is .928 (Ex. 2); therefore, the cube root of $\frac{4}{5}$ is .928+.

Rule. — 1. *Point off the given number into periods of three figures each.*

2. *Find the greatest cube in the first period on the left; place its root on the right, like a quotient in division; sub-*

tract the cube from the period, and to the remainder bring down the next period for a dividend.

3. *For a trial divisor take three times the square of the root already found, considered as tens, and divide the dividend by it. The quotient (or quotient diminished) will be the second figure of the root.*

4. *To this trial divisor, add three times the product of the first figure of the root considered as tens, by the second figure and also the square of the second figure. The sum is the complete divisor.*

5. *Multiply the complete divisor by the last figure of the root; subtract the product from the dividend, and to the remainder bring down the next period for a new dividend.*

6. *Find a new trial divisor as before, and continue the operation in the same manner until all the periods are brought down.*

NOTES. — 1. When the number is an imperfect cube, the operation may be continued to any required number of decimal places in the root by bringing down periods of decimal ciphers (Ex. 2).

2. To extract the cube root of a common fraction: when both terms are perfect cubes, extract the cube root of the numerator and then of the denominator (Ex. 3); when both terms are not perfect cubes, reduce the fraction to a decimal and extract the cube root of the decimal (Ex. 4).

Extract the cube root of:

5.	91125.	45.	**11.**	529475129.	809.
6.	195112.	58.	**12.**	958585256.	986.
7.	912673.	97.	**13.**	14760213677.	2453.
8.	1225043.	107.	**14.**	128100283921.	5041.
9.	13312053.	237.	**15.**	53.157376.	3.76.
10.	102503232	468.	**16.**	.199176704.	.584.

17.	$\frac{216}{343}$.	$\frac{6}{7}$.	**21.**	2.	1.259+.
18.	$\frac{2744}{6859}$.	$\frac{14}{19}$.	**22.**	9.	2.080+.
19.	$\frac{48778}{118638}$.	$\frac{29}{39}$.	**23.**	200.	5.848+.
20.	$5\frac{104}{125}$.	$1\frac{4}{5}$.	**24.**	$9\frac{1}{6}$.	2.092+.

245. Given the solid contents of a cube to find its side (§ 70):

Rule. — *Extract the cube root of the solid contents.*

1. The contents of a cubical cellar are 1953.125 cu. ft. Find the length of one side. 12.5 ft.

2. Sixty-four 3-inch cubes are piled in the form of a cube. What is the length of each side? 1 ft.

3. A cubical box contains 512 half-inch cubes. What are the dimensions of the box inside? 4 in.

4. A cubical excavation contains 450 cu. yd. 17 cu. ft. What are its dimensions? 23 ft.

5. Find the side of a cube equal to a mass 288 ft. long, 216 ft. broad, and 48 ft. high. 144 ft.

6. The side of a cubical vessel is 1 foot. Find the side of another cubical vessel that contains 3 times as much. 17.306+ in.

7. What is the length of one side of a cistern of cubical form, containing 1331 solid feet? 11 ft.

8. A cubical box contains 474552 cu. in. What is the area of one of its sides? $42\frac{1}{4}$ sq. ft.

9. How many square feet of zinc will be needed to line the five sides of a cubical tank containing 1728 cu. ft.? 720 sq. ft.

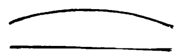
10. Find the length of one side of a cubical marble pedestal containing 373248 cu. in. 6 ft.

MENSURATION

MEASUREMENT OF SURFACES

246. A **line** has length without breadth or thickness.

Lines are either *straight* or *curved*.



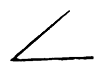
When two lines meet, they form an *angle*.

NOTE. — The point at which the lines meet is called **the vertex** of the angle.

Angles are either *acute*, *obtuse*, or *right* angles.

When two straight lines are perpendicular to each other, they form a **right** angle.

An **acute** angle is less than a right angle.



An **obtuse** angle is greater than a right angle.

When two straight lines are everywhere equally distant they are *parallel*.

A **surface** has length and breadth without thickness.

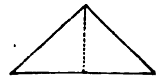
Surfaces are either *plane* or *curved*.

Thus, the surface of a table or a floor is plane; that of a ball or a globe is curved.

A **plane figure** is a portion of a plane surface bounded by straight or by curved lines.

A **polygon** is a plane figure bounded by straight lines.

NOTE. — The straight lines are called the *sides* of the polygon; the **perimeter** of a polygon is the sum of all its sides. The **apothem** of a regular polygon is the perpendicular line drawn from the center of the polygon to the middle of one of its sides.

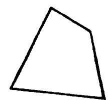


A **triangle** is a plane figure bounded by three straight lines.

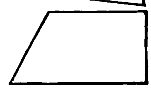
NOTE. — If one side is taken for the base, the perpendicular let fall upon the base from the opposite angle is called the **altitude** of the triangle.

A **quadrilateral** is a plane figure bounded by four straight lines.

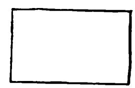
There are three kinds of quadrilaterals: the *trapezium*, the *trapezoid*, and the *parallelogram*.



A **trapezium** is a quadrilateral with no two sides parallel.

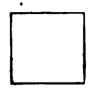


A **trapezoid** is a quadrilateral with only two sides parallel.



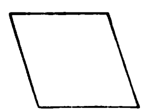
A **parallelogram** is a quadrilateral with its opposite sides equal and parallel.

NOTE. — If one side is taken as the base, the perpendicular let fall upon the base from the opposite side is called the altitude of the parallelogram. Parallelograms may be rectangles (squares or oblongs), rhombuses, or rhomboids.

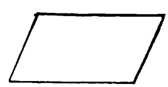


A **rectangle** is a parallelogram with all its angles right angles.

A **square** is a rectangle with all its sides equal.

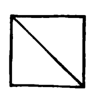


A **rhombus** is a parallelogram with all its sides equal, and its angles not right angles.



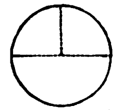
A **rhomboid** is a parallelogram with only its opposite sides equal, and its angles not right angles.

A polygon of five sides is called a **pentagon**; of six, a **hexagon**; of eight, an **octagon**.



A **diagonal** is a line joining two angles not adjacent.

A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the *center*.



The **circumference** of a circle is the curved line that bounds the figure.

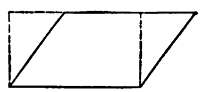
The **diameter** of a circle is a straight line passing through the center, and terminated, both ways, by the circumference.

The **radius** of a circle is a straight line drawn from the center to the circumference; it is half the diameter.

247. To find the area of a parallelogram:

Rule. — *Multiply the base by the altitude.*

EXPLANATION. — The area of a parallelogram is equal to the area of a rectangle, having an equal base and the same altitude; but the area of the rectangle is equal to its length multiplied by its breadth (§ 68); hence, the area of a parallelogram is equal to its base multiplied by its altitude.



1. How many square feet are there in a floor 17 ft. long and 15 ft. wide? 255 sq. ft.

2. How many acres of land are there in a rectangle 120 rd. long and 84 rd. wide? 63 A.

3. How many acres are there in a square field, each side of which is 65 rd.? 26 A. 65 sq. rd.

4. How many acres are there in a field in the form of a rhombus, each side measuring 35 rd., and the perpendicular distance between two sides being 16 rd.? 3 A. 80 sq. rd.

5. Find the difference in area between a floor 30 ft. square, and two others each 15 ft. square. 50 sq. yd.

6. A table is 3 ft. 4 in. long, and 2 ft. 10 in. wide. How many square feet are in its surface?

SOLUTION. — 3 ft. 4 in. = $3\frac{1}{3}$ or $\frac{10}{3}$ ft.; 2 ft. 10 in. = $2\frac{5}{6}$ or $\frac{17}{6}$ ft. The surface of the table is $\frac{10}{3} \times \frac{17}{6} = 9\frac{4}{9}$ sq. ft.

7. How many square feet are there in a marble slab 5 ft. 6 in. long and 1 ft. 8 in. wide? $9\frac{1}{6}$ sq. ft.

8. How many square yards are there in a ceiling 25 ft. 9 in. long, and 21 ft. 3 in. wide?
60 sq. yd. 7 sq. ft. 27 sq. in.

9. A room is 10 ft. long. How wide must it be to contain 80 sq. ft.? 8 ft.

10. How many yards of carpet, $1\frac{1}{2}$ yd. wide, will cover a floor 18 ft. long and 15 ft. wide? 20 yd.

11. How many yards of flannel, $\frac{3}{4}$ yd. wide, will it take to line 3 yd. of cloth, $1\frac{1}{2}$ yd. wide? 6 yd.

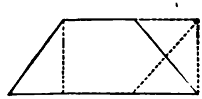
12. How many yards of carpet, $1\frac{1}{4}$ yd. wide, will it take to cover a floor 21 ft. 3 in. long and 13 ft. 6 in. wide? $25\frac{1}{2}$ yd.

13. A rectangular field is 15 rd. long. What must be its width to contain 1 A.? $10\frac{2}{3}$ rd.

248. To find the area of a trapezoid:

Rule. — *Multiply half the sum of the parallel sides by the altitude.*

EXPLANATION. — The base of a parallelogram having the same altitude and an equal area is one half the sum of the parallel sides of the trapezoid.



1. The parallel sides of a trapezoid are 2 ft. 2 in. and 2 ft. 11 in.; its altitude is 11 in. What is its area?
2 sq. ft. $47\frac{1}{2}$ sq. in.

2. A field is in the form of a trapezoid; one of the parallel sides is 25 rd., and the other 19 rd.; the width is 32 rd. How many acres are there in the field?

4 A. 64 sq. rd.

3. How many square yards are there in a piece of roof 10 ft. 8 in. wide on the lower side, and 6 ft. 2 in. wide on the upper side, the length being 12 ft.?

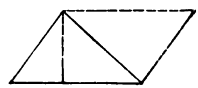
11 sq. yd. 2 sq. ft.

249. To find the area of a triangle:

1st. *When the base and altitude are given:*

Rule. — *Multiply the base by half the altitude.*

Explanation. — The area of a triangle is one half the area of a parallelogram having the same base and altitude.



2d. *When the three sides are given:*

Rule. — 1. *From half the sum of the three sides subtract each side separately.*

2. *Find the product of the half-sum and the three remainders, and extract the square root of this product.*

1. The base of a triangle is 15 ft. and its altitude 12 ft. What is its area? 90 sq. ft.

2. One side of a triangular lot is 44 rd., and the perpendicular distance from the angle opposite to this side is 18 rd. How many acres are in the lot? 2 A. 76 sq. rd.

3. What is the area of a triangle of which the base is 12 ft. 6 in. and the altitude 16 ft. 9 in.?

11 sq. yd. 5 sq. ft. 99 sq. in.

4. Find the area of a triangle whose sides are 13 ft., 14 ft., and 15 ft. 84 sq. ft.

5. The sides of a triangle are 30 ft., 40 ft., and 50 ft. What is the area? 66 sq. yd. 6 sq. ft.

250. To find the area of a trapezium or other irregular figure:

Rule.—1. *Divide the figure into triangles by diagonals.*
2. *Find the areas of the triangles, and add them together.*

1. Find the area of a field in the form of a trapezium, of which a diagonal is 50 rd. and the perpendiculars to the diagonal from the opposite angles 30 rd. and 20 rd.
7 A. 130 sq. rd.

251. To find the area of a regular polygon:

Rule.—*Multiply the perimeter by half the apothem.*

1. Find the area of a regular pentagon whose sides are each 8 in. and apothem 5.44 in. 108.8 sq. in.
2. Find the area of a regular hexagon whose sides are each 3 in. and apothem 2.6 in. 23.4 sq. in.

252. To find the circumference of a circle when the diameter is given:

Rule.—*Multiply the diameter by* 3.1416.

To find the diameter of a circle when the circumference is given:

Rule.—*Divide the circumference by* 3.1416.

1. The diameter of a circle is 48 ft. What is the circumference? 150 ft. 9.56 in.
2. The circumference of a circle is 15 ft. What is the diameter? 4 ft. 9.3 in.
3. The diameter of a wheel is 4 ft. What is its circumference? 12 ft. 6.8 in.
4. If the girth of a tree is 12 ft. 5 in., what is its diameter? 3 ft. 11.43 in.
5. What is the circumference of the earth, the diameter being 7912 mi.? 24856+ mi.

253. To find the area of a circle, when the radius is given:

Rule.—*Multiply the square of the radius by* 3.1416.

To find the radius of a circle when the area is given:

Rule.—*Divide the area by* 3.1416, *and extract the square root of the quotient.*

1. Find the area of a circle whose radius is 21 ft.
153 sq. yd. 8 sq. ft. 64 sq. in.

2. The area of a circle is 6 sq. ft. 98.115 sq. in. What are its diameter and circumference?
2 ft. 11 in.; 9 ft. 1.9+ in.

3. How long a rope will it take to fasten a horse to a post so that he may graze over 1 A. of grass, and no more? 7 rd. 2 ft. 3 in.

4. Two circles, 10 and 16 ft. in diameter, have the same center. What is the area of the ring between their circumferences? 122 sq. ft. 75 sq. in.

5. The area of a circle is 1 square foot. What is its diameter? 13.54 in.

Similar Plane Figures

254. Similar plane figures are such as have the same *form;* or have angles equal each to each, the same number of sides, and the sides containing the equal angles proportional.

NOTE.—All circles, squares, equiangular triangles, and regular polygons of the same number of sides are similar figures. The like dimensions of circles, that is, their radii, diameters, and circumferences are proportional.

Principles.—I. *The areas of similar plane figures are to each other as the squares of their like dimensions.*

II. *The like dimensions of similar plane figures are to each other as the square roots of their areas.*

1. The area of a circle is 10 square inches. What is the area of a circle whose diameter is twice as great?

SOLUTION.—$1^2 : 2^2 :: 10 : ?$ $\frac{4 \times 10}{1} = 40.$ 40 sq. in. *Ans.*

2. The diameter of a circle whose area is $28\frac{1}{4}$ sq. in. is 6 in. Find the diameter of a circle whose area is 113 sq. in.

SOLUTION.—$28\frac{1}{4} : 113 :: 1 : 4.$ $\sqrt{1} : \sqrt{4} :: 6 : ?$ $\frac{6 \times 2}{1} = 12$ in. *Ans.*

3. A circular room has a diameter of 75 feet. Find the diameter of a circular room $\frac{1}{3}$ as large. 43.3 ft.

4. If 63.39 rods of fence will inclose a circular field of 2 A., how many rods will inclose a circular field of 3 A.? 77.63 rods.

5. If the area of a circle is 1 sq. ft., what is the area of a circle with a radius 4 times as large? 16 sq. ft.

6. A circle 30 ft. in diameter is how many times as large as one 6 ft. in diameter? 25 times.

7. If the side of a square field containing 10 A. is 40 rd., find the side of a square field containing 30 A. 69.28 rd.

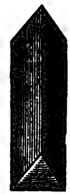
8. Two circles are to each other as 9 to 16. The diameter of the less is 112 feet. Find the diameter of the greater. $149\frac{1}{3}$ ft.

9. The radius of a circular garden bed which has an area of 78.54 sq. ft. is 5 ft. Find the radius of a similar bed having an area of 19.635 sq. ft. $2\frac{1}{2}$ ft.

MEASUREMENT OF SOLIDS

255. A **solid**, or **body**, has length, breadth, and thickness.

A **prism** is a solid with two parallel bases, which are polygons, and with its faces parallelograms.



NOTE. — A prism is triangular, quadrangular, etc., according to the shape of the base.

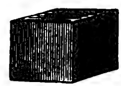
A **right prism** has its faces rectangles.

The **altitude** of a prism is the perpendicular let fall from one base upon the other.

The **convex surface** of a prism is the sum of the areas of its faces.

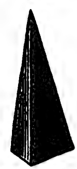
A **parallelopipedon** is a prism with its bases parallelograms.

A **right parallelopipedon** is a solid with six rectangular faces.



A **cube** is a solid with six equal square faces.

A **pyramid** is a solid with one base, which is a polygon, and with its faces triangles.



A **right pyramid** has all its faces equal.

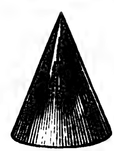
The **slant height** of a right pyramid is the perpendicular from the vertex to the middle of each side of the base.

The three round bodies are the *cylinder*, the *cone*, and the *sphere*.

A **cylinder** is a solid with two parallel bases, which are circles, and with a curved surface.

The **axis** of a cylinder is a line joining the centers of the two bases.

The **convex surface** of a cylinder is the area of its curved surface.



A **cone** is a solid with one base, which is a circle, and with a curved surface terminating in an apex.

A **sphere** is a solid with a curved surface, every point of which is equally distant from a point within called the *center*.

The **volume** of a body is its solid contents.

256. To find the convex surface of a right prism:

Rule. — *Multiply the perimeter of the base by the altitude.*

To find the convex surface of a cylinder:

Rule. — *Multiply the circumference of the base by the altitude.*

To find the entire surface of a prism or of a cylinder:

Rule. — *To the convex surface add the areas of the two bases.*

1. Find the surface of a cube, each side of which is 37 in. 6 sq. yd. 3 sq. ft. 6 sq. in.

2. Find the surface of a right prism, with a triangular base, each side of which is 4 ft., the altitude of the prism being 5 ft. 73.85+ sq. ft.

3. Find the surface of a box that is 3 ft. 6 in. long, 2 ft. 9 in. wide, and 1 ft. 10 in. high. $42\frac{1}{6}$ sq. ft.

4. Find the surface of a cylinder, its altitude being 5 ft. and the radius of the base 2 ft. 87.96+ sq. ft.

257. To find the volume of a prism or of a cylinder:

Rule. — *Multiply the area of the base by the altitude.*

Note. — The rule for finding the volume of a right parallelopipedon is given in § 70.

1. Find the volume of a right parallelopipedon, of which the length is 12 ft., the width 3 ft. 3 in., and the height 4 ft. 4 in. 169 cu. ft.

SOLUTION. — 3 ft. 3 in. = $3\frac{1}{4}$ ft. or $\frac{13}{4}$ ft.; 4 ft. 4 in. = $4\frac{1}{3}$ ft. or $\frac{13}{3}$ ft. Therefore, the volume of the parallelopipedon is $12 \times \frac{13}{4} \times \frac{13}{3} = 169$ cu. ft.

2. How many cubic yards are there in a room 24 ft. long, 18 ft. 6 in. wide, and 10 ft. 7 in. high?
174 cu. yd. 1 cu. ft.

3. Each side of the base of a triangular prism is 2 ft.; its altitude is 14 ft. What is the volume of the prism?
$24\frac{1}{4}$ cu. ft. nearly.

4. Find the volume of a cylinder whose altitude is 12 ft. and the radius of the base 2 ft. 150.8 cu. ft. nearly.

5. How many cubic inches are there in a peck measure, the diameter of the bottom being $9\frac{1}{4}$ in. and the depth 8 in.? 537.6+ cu. in.

258. To find the convex surface of a right pyramid:

Rule. — *Multiply the perimeter of the base by the slant height, and take half the product.*

To find the convex surface of a cone:

Rule. — *Multiply the circumference of the base by the slant height, and take half the product.*

To find the entire surface of a pyramid or of a cone:

Rule. — *To the convex surface add the area of the base.*

1. Find the entire surface of a right pyramid, with a triangular base, each side of which is 5 ft. 4 in., the slant height of the pyramid being 7 ft. 6 in. 72.3+ sq. ft.

2. What is the convex surface of a cone of which the slant height is 25 ft. and the diameter of the base 8 ft. 6 in.? 333.8 sq. ft. nearly.

3. Find the entire surface of a cone of which the slant height is 4 ft. 7 in. and the diameter of the base 2 ft. 11 in. 27.6+ sq. ft.

259. To find the volume of a pyramid or of a cone:

Rule. — *Multiply the area of the base by the altitude, and take one third of the product.*

1. Find the volume of a square pyramid of which each side of the base is 5 ft. and the altitude 21 ft. 175 cu.ft.

2. Find the volume of a cone of which the altitude is 15 ft. and the radius of the base 5 ft. 392.7 cu. ft.

3. A square pyramid is 477 ft. high; each side of its base is 720 ft. How many cubic yards are there in the pyramid? 3052800 cu. yd.

4. The diameter of the base of a conical glass house is 37 ft. 8 in., and its altitude 79 ft. 9 in. What is the space inclosed? 29622+ cu. ft.

260. To find the surface of a sphere:

Rule. — *Multiply the square of the diameter by* 3.1416.

1. What is the surface of a sphere of which the diameter is 1 ft.? 3.14+ sq. ft.

2. What is the surface of a sphere of which the diameter is 4 ft. 6 in.? 63.6+ sq. ft.

3. What is the area of the earth's surface, on the supposition that it is a perfect sphere 7912 miles in diameter? 196663355.75+ sq. mi.

261. To find the volume of a sphere:

Rule. — *Multiply the cube of the diameter by one sixth of* 3.1416, *or* .5236.

1. Find the volume of a sphere 13 ft. in diameter. 1150.3+ cu. ft.

2. The volume of a sphere is 1 cu. ft. What is its diameter? 14.9 in. nearly.

Similar Solids

262. Similar solids have the same form and differ from each other only in volume.

Principles. — I. *The volumes of similar solids are to each other as the cubes of their like dimensions.*

II. *The like dimensions of similar solids are to each other as the cube roots of their volumes.*

1. If the volume of a 3-inch cube is 27 cu. in., what is the volume of a 7-in. cube?

SOLUTION. — $3^3 : 7^3 :: 27 : ?$ $\frac{\cancel{27} \times 343}{\cancel{27}} = 343$ cu. in. *Ans.*

2. If the side of a cube containing 27 cu. in. is 3 in., what is the side of a cube containing 343 cu. in?

SOLUTION. — $\sqrt[3]{27} : \sqrt[3]{343} :: 3 : ?$ $\frac{\sqrt[3]{343} \times 3}{\sqrt[3]{27}} = \frac{7 \times \cancel{3}}{\cancel{3}} = 7$ in. *Ans.*

3. If a rectangular solid 16 in. long contains 512 cu. in., find the length of a similar solid containing 64 cu. in. 8 in.

4. The volume of a cone whose altitude is 8 in. is 50.2656 cu. in. Find the volume of a similar cone whose altitude is 12 in. 169.6464 cu. in.

5. A ball weighs 32 lb. Find the weight of a ball of the same material with half the diameter of the first. 4 lb.

EXAMPLES FOR REVIEW

263. **1.** Four men engaged in trade; A put in \$ 1275, and B \$ 1350; C put in \$ 2580 more than B, and D put in as much as A and B together. How much did all put in? \$ 9180.

2. A man traveled 6784 miles; 2324 miles by railroad, 1570 miles in a stagecoach, 450 miles on horseback, 175 miles on foot, and the remainder by steamboat. How many miles did he travel by steamboat? 2265 miles.

3. Two trains leave a station at the same time and travel in the same direction, the first 38 miles an hour, and the second 30 miles an hour. How far apart will they be at the end of two days? 384 mi.

4. A factory made 16150 boxes on Monday, 17050 on Tuesday, 17364 on Wednesday, 17500 on Thursday, 18008 on Friday, and 18169 on Saturday. The week following 9000 more boxes were made than during the preceding week. How many boxes were made in the two weeks? 217482.

5. To allow 50 cubic feet of air per minute for each person, how many cubic feet of air per hour should pass through an assembly room containing 560 people? 1680000 cu. ft.

6. In a certain year the expenses of a government were \$ 356195298. The revenues of the government for that year were \$ 313390075. How much did the amount spent exceed the amount received? \$ 42805223.

7. Commodore Dewey's fleet consisted of the *Olympia*, 5800 tons; *Baltimore*, 4600 tons; *Boston*, 3189 tons; *Raleigh*, 3182 tons; *Concord*, 1700 tons; *Petrel*, 890 tons. What was the total tonnage of the fleet? 19361 T.

8. A forest in Maine contains 3000 trees, of which one in every 15 is a balsam. If the tourists cut down all the balsam trees, how many trees will be left? 2800.

9. If one mile of railroad requires 116 tons of iron, worth $ 65 a ton, what will be the cost of sufficient iron to construct a road 128 miles in length? $ 965120.

10. A farm consists of 7 fields, containing $12\frac{3}{4}$ acres, $18\frac{2}{5}$ acres, 9 acres, $24\frac{1}{8}$ acres, $4\frac{13}{16}$ acres, $8\frac{9}{10}$ acres, and $15\frac{13}{20}$ acres. How many acres are there in the farm? 93.6375 A.

11. How many bushels of corn, worth 55 cents a bushel, must be given in exchange for 3 pieces of cloth, each containing 33 yards, at 25 cents a yard? 45 bu.

12. A farmer had $\frac{1}{5}$ of his sheep in one pasture, $\frac{1}{4}$ in another, and the remainder, which were 77, in a third pasture. How many sheep had he? 140 sheep.

13. A man bought 240 acres of land at $ 15 an acre, giving in payment horses valued at $ 180 apiece. How many horses did he give? 20.

14. How much will 963 bushels of oats cost at $33\frac{1}{3}$ cents per bushel? $ 321.

15. I paid $ 36 an acre for 50 acres of woodland. I sold the wood for $ 1576, and the land for $ 17 an acre. Did I gain or lose, and how much? $ 626 gain.

16. If a horse eats $\frac{3}{8}$ of a bushel of oats in a day, in how many days will he eat $5\frac{1}{4}$ bushels? 14 days.

17. A merchant sold his stock for $ 2000, which was at a loss of 10%. How much did the stock cost? $ $2222\frac{2}{9}$.

18. A grocer sold 18 boxes of soap, each containing 55 pounds, at 10 cents a pound, and received as pay 66 barrels of apples, each containing 3 bushels. What was the price per bushel of the apples? 50 ¢.

19. A man, having $271\frac{1}{2}$ acres of land, sold $\frac{1}{3}$ of it, and gave $\frac{3}{8}$ of it to his son. What was the value of the remainder, at $ 57 per acre? $ 4513.6875.

20. A man's income from an investment is \$4000, which is 15% of the sum invested. How much has he invested?

$ 26666⅔.

21. If a rod 4 feet long casts a shadow $6\frac{3}{8}$ feet long, what is the length of the shadow that a rod $36\frac{1}{8}$ feet long will cast at the same time of day? $60\frac{5}{24}$ ft.

22. A person after spending \$40 more than .6 of his money had \$60 less than $.42\frac{6}{7}$ of it left. How much money had he at first? \$700.

23. A grocer after selling $\frac{1}{8}$, $\frac{1}{10}$, $\frac{1}{5}$, and $\frac{1}{4}$ of a quantity of sugar had 260 pounds left. How many pounds had he at first?

800 lb.

24. What is the cost of 1250 yd. of silk at $87\frac{1}{2}$¢ per yard?

\$1093.75.

25. Twenty-three miles of a railroad, 47.95 miles long, cost \$11578.40 per mile; 12 miles cost \$13357.82 per mile, and the remainder cost \$19125.26 per mile. What was the average cost per mile of the entire road? \$14061.92 +.

26. How much will it cost to pave a walk, 60 ft. long and 15 ft. wide, at \$1.25 a square yard? \$125.

27. Three men engage to do a piece of work. A can do it in 12 days, B in 15 days, and C in 18 days. In what time can they do it together? $4\frac{32}{37}$ days.

28. At $16\frac{2}{3}$ cents a dozen, how much do 1935 dozen eggs cost? \$322.50.

29. I loaned a man a certain sum of money; at one time he paid me \$59.75, which was $12\frac{1}{2}$% of the whole sum loaned to him. How much did I loan him? \$478.

30. The distance around a circle is about 3.1416 times the distance across it. If the distance across a certain circular race course is 1710 feet, what is the distance around it?

5372.136 ft.

31. What per cent of 15 pounds is 5 pounds 10 ounces, avoirdupois weight? $37\frac{1}{2}$%.

32. A broker in New York exchanged $ 25875 on the Suffolk Bank, Boston, at $\frac{1}{8}\%$. How much brokerage did he receive? $ 32.34.

33. A girl wishes to divide two pieces of tape, one 42 ft. long, the other 56 ft., into pieces of equal length, each as long as possible. How long will each piece be after this division, and how many pieces will there be? 14 ft., 7 pieces.

34. If a grocer mixes 8 lb. of tea worth $ .60 a pound with 6 lb. at $ .70 a pound, 2 lb. at $ 1.10, and 4 lb. at $ 1.20, what is 1 lb. of the mixture worth? $ .80.

35. If a man working $8\frac{1}{2}$ hours a day can finish a piece of work in 12 days, how many hours per day must he work to complete it in $8\frac{3}{4}$ days? $11\frac{23}{35}$ hr.

36. Two men engaged in trade, each with $ 2760. One of them gained $33\frac{1}{3}\%$ of his capital, and the other gained 75%. How much more did the one gain than the other? $ 1150.

37. Mr. Brown earned $ 2.00 for each day of 10 hours that he worked. On Monday he worked 8 hr. 30 min.; Tuesday, 9 hr. 10 min.; Wednesday, 7 hr. 40 min.; Thursday, 8 hr. 30 min.; Friday, 7 hr. 50 min.; Saturday, 4 hr. 30 min. How much did he earn in that week? $ 9.23.

38. What must be the length of a bin 1 meter wide and 1 meter deep, to contain 4500 liters of grain? 4.5^{m}.

39. A milkman sold one morning 220 qt. of milk at 6 ¢ a quart. His measure lacked $\frac{1}{5}$ of a gill of holding a full quart. What was the actual worth of the milk sold? $ 12.87.

40. Find the proceeds of a note for $ 368, at 90 days, discounted at a bank at 6%. $ 362.48.

41. Dorr & Co., piano dealers, select from the manufacturer's catalogue 3 pianos listed at $ 400, $ 450, and $ 700, and order 6 pianos of each kind. The trade discount is 60, 10, and 5. What is the cost of that shipment of pianos? $ 3180.60.

42. If I pay $ 45 interest for the use of $ 500 for 3 years, what is the rate? 3%.

43. Two brothers together own $\frac{1}{3}$ of a flouring mill valued at \$ 12520. A owns $\frac{3}{7}$ as much as B. What is the value of each one's share? A's \$ 1252; B's \$ 2921$\frac{1}{3}$.

44. An agent buys for a manufacturing company 26750 pounds of wool, at 32 cents a pound, and receives a commission of 2$\frac{3}{4}$%. What amount does he receive? \$ 235.40.

45. A man owns a house from which he receives a monthly rental of \$ 25. The insurance on it for the year 1903 was \$ 8 and the taxes were \$ 36.75. It was vacant three months. How much was his net income from it? \$ 180.25.

46. A man bought a piece of property for \$ 2870 and agreed to pay for it in 1 year and 6 months, with 6$\frac{1}{2}$% interest. What amount did he owe? \$ 3149.825.

47. A bill of goods at list prices amounted to \$ 420.65. The discounts were 25% and 10%. What was due on the bill?

\$ 283.94.

48. If the consequent is 13$\frac{3}{7}$ and the ratio 8$\frac{1}{6}$, what is the antecedent? 109$\frac{2}{3}$.

49. Find the cost and the marked price of goods sold at \$ .75, which was a reduction of 25% from their marked price and an advance of 50% upon their cost.

Cost \$ .50; marked price, \$ 1.00.

50. One half the sum of two numbers is 800, and one half the difference of the same numbers is 200. What are the numbers? 1000 and 600.

51. A man sold two houses for \$ 3600 each. On one he gained 25%, and on the other he lost 25%. How much was gained or lost by the transaction? \$ 480 loss.

52. A fisherman has four boats. One will carry 8 barrels, another 9, another 15, and another 16. What is the smallest number of barrels that will make full freight for any one and all of the boats? 720.

53. A lent B \$ 1200 until the interest at 6% amounted to \$ 720. How long was the money loaned? 10 years.

54. The top of a castle is 45 yards high, and the castle is surrounded by a ditch 60 yards wide. What would be the length of a rope that would reach from the outside of the ditch to the top of the castle? 75 yards.

55. Three men trade in company; A furnishes \$ 8000, B \$ 12000, and C \$ 20000 of the capital; their gain is \$ 1680. What is each man's share of the gain?

A's, \$ 336; B's, \$ 504; C's, \$ 840.

56. If a staff 3 ft. 8 in. long casts a shadow 1 ft. 6 in., what is the height of a steeple that casts a shadow 75 feet at the same time? 183 ft. 4 in.

57. A tax of \$ 13662 is to be assessed on a village; the property is valued at \$ 1400000, and there are 2981 polls, to be taxed \$ 2.00. What is the assessment on a dollar? What is C's tax, his property being assessed at \$ 12450 and he paying for 2 polls? \$ .005½ on \$ 1; \$ 72.47½, C's tax.

58. At the rate of 36^{Km} per hour, how far will a train run in 3 hr. 30 min.? 126^{Km}.

59. If the antecedent is \$ 6.12½ and the ratio 25, what is the consequent? \$ .245.

60. What is the rate of income upon money invested in 6% bonds, purchased at 87? $6\frac{26}{29}\%$.

61. A merchant bought 360 barrels of flour, and after losing $66\frac{2}{3}\%$ of it, he sold $87\frac{1}{2}\%$ of the remainder. What per cent of the whole had he left? $4\frac{1}{6}\%$.

62. A farmer used $77\frac{1}{2}$ acres of land for wheat, $40\frac{7}{8}$ acres for corn, $1\frac{5}{8}$ acres for vegetables, $29\frac{1}{2}$ acres for pasturage, and $10\frac{1}{4}$ acres for an orchard. How many acres were there in his farm? $159\frac{3}{4}$ A.

63. How many silver spoons, each weighing 2 oz. 5 pwt., can be made from a bar of silver weighing 6 lb. 4 oz. 10 pwt.? 34 spoons.

64. At what price must 4% stock be bought to pay as good an income as 8% stock bought at par? 50.

65. How large a draft at sight on London can be bought in Chicago for \$ 1950, when exchange is \$ $4.86\frac{2}{3}$?

£ 400 13 *s.* 8 *d.* +.

66. When taxes are \$ 16 on \$ 1000, how much is a person taxed who has \$ 13000 worth of real estate and \$ 28000 of personal property? \$ 656.

67. A, B, and C agree to build a house. A and B can do the work in 32 days, B and C in 28 days, and A and C in 26 days. How long will it take them to do it working together? How long will it take each to do it alone?

All $18\frac{298}{307}$ da.; A $58\frac{82}{99}$ da.; B $70\frac{14}{83}$ da.; C $46\frac{74}{125}$ da.

68. A grain dealer paid \$ 225 for insuring a cargo of wheat at $1\frac{1}{2}\%$. For how much was it insured? \$ 15000.

69. If a note of \$ 605.70, given May 20, 1900, on interest at 8%, is taken up May 20, 1903, what amount will then be due if no interest has been paid? \$ 751.07.

70. If from a string 200 yd. 2 ft. long, $2.87\frac{1}{2}$ ft. is broken off at one end, and $3.12\frac{1}{2}$ ft. at the other, how long a string is left?

596 ft.

71. How long must \$ 204 be on interest at 7%, to amount to \$ 217.09? 11 months.

72. What will be the cost of a board 20 feet long, 22 in. wide at one end and 16 in. at the other, and $1\frac{1}{2}$ in. thick, at \$ 25 per M? \$ 1.19.

73. Two ships sail from the same port; one goes due north 128 miles, and the other due east 72 miles. How far are the ships from each other? 146.86 + miles.

74. What was the list price of an article whose net cost was \$ 4.50, after deducting discounts of 40% and 10%? \$ $8.33\frac{1}{3}$.

75. An agent sold goods to the amount of \$ 1260. What was his commission at $3\frac{1}{2}\%$? \$ 44.10.

76. If 15 men can do a piece of work in 36 days, in how many days can they perform the same work with the assistance of 9 men more? $22\frac{1}{2}$ da.

77. Find the compound interest of \$500 for 2 years 6 months at 6%. \$78.654.

78. What number is that which being increased by its half, its third, and 18 more, will be doubled? 108.

79. A furniture dealer paid \$76 for tables and chairs, paying \$3 for tables and \$2 for chairs; he sold 3 of his tables and $\frac{2}{5}$ of his chairs for \$23, and by so doing lost 8% on their cost. How many of each did he purchase? 20 chairs, 12 tables.

80. The shadow of a tree measures 42 ft.; a staff 40 in. in length casts a shadow 18 in. at the same time. What is the height of the tree? $93\frac{1}{3}$ ft.

81. Two men start from one corner of a park one mile square, and travel at the same rate. A goes by the walk around the park, and B takes the diagonal path to the opposite corner, and turns to meet A at the side. How many rods from the corner will the meeting take place? 93.7 + rods.

82. A farmer had a field 21 rd. square. Three rows of wire fencing were put around it, costing 2¢ a foot, 5% off for cash. If he paid cash, how much did the fence cost? \$79.

83. The taxable property in a certain town is valued at \$1360000, and a tax of \$8840 is voted for school purposes. What is the rate of taxation? .0065.

84. What sum must be invested in U. S. 4's at $121\frac{1}{4}$, brokerage at $\frac{1}{8}$%, to secure an annual income of \$900? \$27309.38.

85. If I invest \$867 in 6% bonds at 102, what income will I receive on my investment? \$51.

86. How many meters of fence would be required to inclose a hectare in the form of a square? 400 ^{m}.

87. A grocer has a false balance, by which he sells 12 oz. for a pound. What is the real value of a barrel of sugar that sells for \$28? \$21.

88. B owned 75 shares of stock in a building association, at \$50 each. The association declared a dividend of 8%, payable in stock. How many shares did he then own? 81 shares.

89. What is the volume of a sphere 39 ft. in diameter if the volume of a sphere 13 ft. in diameter is 1150.3 cu. ft.?
31058.1 cu. ft.

90. A, B, and C in partnership gained $ 3192. A's stock was $ 5600, which was $1\frac{1}{3}$ times B's, and B's was $1\frac{1}{5}$ times C's. What was the gain of each?
$ 1344 A's, $ 1008 B's, $ 840 C's.

91. Find approximately the number of kilometers in 235 mi.
376$^{\text{Km}}$.

92. A bin that is just twice as long as it is wide or high holds 500 bushels. What is its length? 13.55 ft.

93. What income will a man derive from $ 10777.375 invested in railroad bonds paying an annual dividend of 10%, if he buys them at $98\frac{3}{4}$, brokerage $\frac{1}{8}$%? $ 1090.

94. How much is gained or lost by buying 440 lb. of dried fruit at 10 ¢ a pound and selling it at 22 ¢ a kilogram, counting $2\frac{1}{5}$ pounds to the kilogram? Nothing.

95. I sent my agent $ 53500 to be invested in Illinois Central Railroad stock, which was selling at 107. What amount did he purchase? $ 50000.

96. How far from the base of a building must a ladder 50 feet long be placed to reach a window 40 feet from the ground?
30 ft.

97. What per cent of income does stock paying 10% dividends yield, if bought at 106? $9\frac{23}{53}$%.

98. A man was offered $ 3675 in cash for his house, or $ 4235 in 3 years, without interest. He accepted the latter offer. How much did he lose, money being worth 7%? $ 175.

99. What must I pay for a 6% stock so that the investment may yield 5%? 120.

100. A cistern with a capacity of 200 gallons can be filled by one pipe in 15 minutes and emptied by another pipe in 40 minutes. If the cistern is empty and both pipes are opened at once, how long will it take to fill it? 24 min.

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